E 1600 (Pages : 3) Reg. No.....

Name.....

C.B.C.S.S. – B.Sc. DEGREE EXAMINATION, APRIL 2011

Fourth Semester

Core Course—VECTOR CALCULUS, THEORY OF EQUATIONS AND NUMERICAL METHODS

(For Model I and Model II B.Sc. Mathematics and B.Sc. Computer Applications)

Time : Three Hours Maximum Weight : 25

Part A (Objective Type Questions)

Answer **all** *questions.*
Each bunch of 4 questions has weight 1.

I. 1 Write a parameteric equation for the line through P(–3, 2, – 3) and Q (1, – 1, 4).

2 If the plane through P(3, 4, – 1) normal to the vector $n = i - j + k$ has an equation $x - 2y + 3 = D$, then what is D ?

3 Find the intercept of $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z}{c}$ on the axes.

4 Give a point of discontinuity of the vector function $g(t) = (\cos t)i + (\sin t)\, j + [t]k$, where $[t]$ is the greatest integer function.

II. 5 Find the arc length parameter along the helix $r(t) = (\cos t)i + \sin t\, j + tk$ from point $t_0 = 0$.

6 Find the gradient field of $f(x, y, z) = xyz$.

7 State whether the field $F = (2x - 3)\, i - zj + (\cos z)k$ is conservative.

8 Find the divergence of $F = (x, y) = \left(x^2 - y\right)i + \left(xy - y^2\right)j$.

III. 9 Find a parametrization fo the cone $z = \sqrt{x^2 + y^2}$, $0 \le z \le 1$.

10 Find the curl of $F = xyi + zj + y^2k$.

11 If α, β, γ are the roots of $2x^3 + x^2 - 2x - 1 = 0$, then what is the value of $\beta\gamma + \gamma\alpha + \gamma\beta$?

12 Give an example of an equation for which $\alpha = 2$ and $\beta = 3$ are 3-multiple roots.

Turn over

IV. 13 If $a_0 x^n + a_1 x^{n-1} + \cdots + a_{n-1} x + a_n = 0$, then what are the possible values of $\frac{a_0}{a_n}$?

14 If a cubic equation $ax^3 + 3bx^2 + 3cx + d = 0$ is written in the form $y^3 + \frac{3H}{a^2} y + \frac{G}{a^3} = 0$, then what is the value of H ?

15 Find two numbers a and b such that a real root of $f(x) = x^3 - x - 1 = 0$ lies between a and b.

16 In Newton-Raphson method to find the real root of an equation $f(x) = 0$, what we are replacing the part of the curve between $(x_0, f(x_0))$ and the X-axis, where x_0 is the initial approximation to a root ?

(4 x 1 = 4)

Part B (Short Answer Type Questions)

Answer any **five** *questions.*
Each questions has weight 1.

17 Find the unit tangent vector of the curve $r(t) = (2 \cos t)i + (2 \sin t)j + \sqrt{5}\, t\, k$.

18 Find the directions in which $f(x, y) = \left(\frac{x^2}{2}\right) + \left(\frac{y^2}{2}\right)$ increases and decreases most rapidly at the point (1, 1).

19 Evaluate $f(x, y, z) = 3x^2 - 2y + z$ over the line segment C joining the origin to the point (2, 2, 2).

20 Find the work done by the conservative field $F = yzi + xzj + xyk$ along any smooth curve C joining the point A(– 1, 3, 9) to B (1, 6, – 4).

21 Calculate the outward flux of the field $F(x, y) = x^2 i + xyj$ across the square bounded by the lines $x = 0$, $y = 0$, $x = a$ and $y = a$, where $a > 0$ using Green's theorem.

22 Solve the equation $x^4 - 8x^3 + 17x^2 - 8x + 1 = 0$.

23 Trasform $x^3 - 6x^2 + 5x + 12 = 0$ into an equation lacking the second term.

24 Write the condition for the sequence of approximations to a real root of an equation $f(x) = 0$ converges to the required root in the method of iteration.

(5 x 1 = 5)

Part C (Short Essay Type Questions)

Answer any **four** *questions.*
Each question has weight 2.

25 The surfaces $f(x, y, z) = x^2 + y^2 - 2 = 0$ and $g(x, y, z) = x + z - 4 = 0$ meet in an ellipse E. Find parametric equations for the line tangent to E at the point P_0 (1, 1, 3).

26 Find a potential function f for the field $\mathbf{F} = 2xi + 3yj + 4zk$.

27 Integrate $g(x, y, z) = x + y + z$ over the surface of the cube cut from the first octant by the planes $x = a$, $y = a$, $z = a$.

28 If α, β, γ are the roots of $x^3 + qx + r = 0$, find the equation whose roots are :

$(\beta - \gamma)^2, (\gamma - \alpha)^2, (\alpha - \beta)^2$.

29 Find to five places of decimals a real root of :

$x^3 + 29x - 97 = 0$.

30 Find a real root of the equation $\cos x = 3x - 1$ using iterative method.

$(4 \times 2 = 8)$

Part D (Essay Type Questions)

Answer any **two** *questions.*
Each question has weight 4.

31 Find the area of the surface cut from the paraboloid $x^2 + y^2 - z = 0$ by the plane $z = 2$.

32 Prove that every polynomial equation of the n^{th} degree has n and only n roots.

33 Obtian a root to 3 decimal places of $x^5 + 5x + 1 = 0$ using Newton-Raphson method.

$(2 \times 4 = 8)$