E 2141 (Pages : 4) Reg. No.................................

Name.....................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, APRIL 2016

Fourth Semester

Core Course—VECTOR CALCULUS, THEORY OF EQUATIONS AND NUMERICAL METHODS

(Common for Mathematics Model I, II and B.Sc. Computer Applications)

[2013 Admission onwards]

Time : Three Hours Maximum Marks : 80

Part A

Answer **all** *the questions.*
Each question carries 1 mark.

1. Find the parametric equation of the x-axis.

2. Write the formula for the length of a smooth curve $r(t) = f(t)i + g(t)j + h(t)k, a \le t \le b$ that is traced exactly once as t increases from a to b.

3. Find T for the curve :

   $r(t) = (\text{lnsect})\, i + tj, -\pi/2 < t < \pi/2.$

4. State Stoke's theorem.

5. Find the curl of $F(x, y) = \left(x^2 - y\right)i + \left(xy - y^2\right)j.$

6. Form an equation whose roots are the reciprocals of the roots of $x^3 - 6x^2 + 8x - 9 = 0.$

7. State fundamental theorem of Algebra.

8. If α, β, γ are the roots of $x^3 + px^2 + qx + r = 0$, express the value of $\Sigma \frac{1}{\alpha}$.

9. Give an example of a transcedental function.

10. Define a polynomial.

(10 × 1 = 10)

Turn over

Part B

Answer any **eight** *questions.*
Each question carries 2 marks.

11. Find a vector parallel to the line of intersection of the planes $3x-6y-2z=15$ and $2x+y-2z=5$.

12. Find the curvature of the curve $r(t)=(6\sin 2t)i+(6\cos 2t)j+5tk$.

13. Write the equation of hyperbolic paraboloid. Describe the sections cut out by the co-ordinate planes.

14. A coil spring lies along the helix $r(t)=(\cos 4t)i+(\sin 4t)j+tk, 0\le t\le 2\pi$. The spring's density is a constant $\delta=1$. Find the spring's mass and its center of mass and it moment of inertia about the z-axis.

15. Find the work done by the force F from (0, 0, 0) to (1, 1, 1) over the path C given by :

    $r(t)=ti+t^2j+t^4k, 0\le t\le 1$ where $\mathrm{F}=xyi+yzj+xzk$.

16. Test whether $\mathrm{F}=\left(e^x\cos y\right)i-\left(e^x\sin y\right)j+2k$ conservative.

17. Verify normal form of Green's theorem for the field $\mathrm{F}(x,y)=(x-y)i+xj$ and the region $\mathbb{R}$ bounded by the unit circle $r(t)=(\cos t)i+(\sin t)j, 0\le t\le 2\pi$.

18. Solve the equation $81x^3-18x^2-36x+8=0$ whose roots are in harmonic progression.

19. Remove the second term from the equation $x^3-6x^2+4x-7=0$.

20. If α,β,γ be the roots of $x^3+3x^2+2x+1=0$. Find the value of $\sum\alpha^3$.

21. Set up a Newton-Raphson iteration formula for computing the square root of a given positive number.

22. Find a real root of $x^3-x-1=0$ by bisection method.

(8 × 2 = 16)

Part C

Answer any **six** *questions.*
Each question carries 4 marks.

23. Find the parametric equation for the line tangent to the curve of intersection of the surfaces :

    $x+y^2+2z=4, x=1$ at $(1,1,1)$.

24. Show that $yzdx + xzdy + xydz$ is exact and evaluate $\int_{(1,1,2)}^{(3,5,0)} yzdx + xzdy + xydz$.

25. Find a potential function f for the field $F = 2xi + 3yj + 4zk$.

26. Use Green's theorem to find the area of the region enclosed by the ellipse :

$r(t) = (a\cos t)i + (b\sin t)j, 0 \le t \le 2\pi$.

27. Find the area of the surface cut from the bottom of the paraboloid $x^2 + y^2 - z = 0$ by the plane $z = 4$.

28. If $\alpha, \beta, \gamma, \delta$ are the roots of $x^4 - 2x^3 + 2x^2 + 1 = 0$ form the equation whose roots are :

$2 + \frac{1}{\alpha}, 2 + \frac{1}{\beta}, 2 + \frac{1}{\gamma}, 2 + \frac{1}{\delta}$ and hence evaluate $(2\alpha + 1)(2\beta + 1)(2\gamma + 1)(2\delta + 1)$.

29. Solve the equation $6x^5 + 11x^4 - 33x^3 - 33x^2 + 11x + 6 = 0$.

30. Find the sum of the fourth powers of the roots of the equation $x^4 - 5x^3 + x - 1 = 0$.

31. Use the method of false position to obtain a root correct to three decimal places, the equation :

$x^3 + x^2 + x + 7 = 0$.

(6 × 4 = 24)

Part D

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) Find the flux of the field $F = 2xi + (x - y)j$ across the circle $r(t) = (a\cos t)i + (a\sin t)j, 0 \le t \le 2\pi$.

(b) Integrate $g(x, y, z) = x + y + z$ over the surface of the cube cut from the first octant by the planes $x = a, y = a, z = a$.

33. (a) Use Stoke's theorem to evaluate $\int_C F \cdot dr$ if $F = xzi + xyj + 3xzk$ and C is the boundary of the portion of the plane $2x + y + z = 2$ in the first octant traversed counter clockwise.

(b) Find the center of mass of a thin shell of constant density δ cut from the cone $z = \sqrt{x^2 + y^2}$ by the planes $z = 1$ and $z = 2$.

Turn over

34. (a) Solve by Cardan's method :

$$x^3 + x^2 - 9x + 12 = 0 \cdot$$

(b) Solve by Ferravi's method :

$$x^4 + 6x^3 + 14x^2 + 22x + 5 = 0 \cdot$$

(c) Solve $x^3 - 9x^2 + 14x + 24 = 0$, two of whose roots being in the ratio 3 : 2.

35. (a) Use iteration method to find, correct to four significant figures, a real root of $\cos x = 3x - 1$.

(b) Use Newton-Raphson method to obtain a root, correct to three decimal places $x - \cos x = 0$.

(2 × 15 = 30)