:6 (Pages : 3) Reg. No..........

Name.............

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2017

Fourth Semester

Course—VECTOR CALCULUS, THEORY OF EQUATIONS AND NUMERICAL METHODS

Common for B.Sc. Mathematics Model I, II and B.Sc. Computer Applications)

[2013 Admission onwards]

hree Hours Maximum Marks : 80

Part A

Answer **all** *questions.*
Each question carries 1 *mark.*

Find the parametric equation of the parabola $y^2 = 4ax$.

Find the angle between the vectors $2\hat{i} + 4\hat{j} + 4\hat{k}$ and $2\hat{i} + 0\hat{j} + 0\hat{k}$.

State Gauss's Divergence theorem.

Find the curl of $\vec{F} = xyz\left(x\hat{i} + y\hat{j} + z\hat{k}\right)$.

State Descarte's rule of signs.

State fundamental theorem of algebra.

Describe the method of False position.

Give an example of a transcendental function.

State factor theorem.

State a formula for finding the positive square root of a natural number N.

(10 × 1 = 10)

Part B

Answer any **eight** *questions.*
Each question carries 2 marks.

If **f** and **g** are differentiable functions, then prove that $div\,(f \times g) = \mathbf{g}.curl\mathbf{f} - \mathbf{f}.curl\mathbf{g}$.

Evaluate $\int \vec{F}.dr$, where $F = xy\hat{i} + yz^2\hat{j} - y^2z\hat{k}$ from origin to the point (1, 1, 1) along the curve $x = t^2, y = t^3, z = t^4$.

Turn over

13. Find a unit normal vector to the surface $x^2 + y^2 + z^2 = 36$.

14. Find the number and position of the real roots of the equation $x^3 - 3x + 1 = 0$.

15. Solve the equation $x^4 + x^2 - 2x + 6 = 0$, given that $1 + i$ is a root.

16. Find the number of positive real roots of the equation $x^4 - 6x^3 + 10^2 - 6x + 1 = 0$.

17. Solve the reciprocal equation $6x^4 + 35x^3 + 62x^2 + 35x + 6 = 0$.

18. How many real roots are there for the equation $x^7 + x^4 + 10x^3 - 28 = 0$.

19. If α, β, γ are the roots of the equation $x^3 + 3x + 5 = 0$ then find $\Sigma\alpha$, $\Sigma\alpha\beta$, and $\alpha\beta\gamma$.

20. Solve the equation $x^3 - 9x + 1 = 0$ for the root lying between 2 and 3, correct to three sign digits.

21. Find the square root of 8.

22. Solve the equation $x \tan x = -1$, by the method of false position starting with 2.5 and 3.0 initial approximations.

(8 × !

Part C

Answer any **six** *questions.*
Each question carries 4 marks

23. If $\nabla\varphi = (x^2 + yz)\hat{i} + (y^2 + zx)\hat{j} + (z^2 + xy)\hat{k}$, find φ.

24. Prove that the necessary and sufficient condition for the vector $\bar{u}(t)$ to have constant mag is that $\bar{u} \cdot \frac{d\bar{u}}{dt} = 0$.

25. Find the flux of $F = (x - y)\hat{i} + x\hat{j}$ across the circle $x^2 + y^2 = 1$ in the xy-plane.

26. Find the area of the region in the first quadrant within the cardioid $r = a(1 - \cos\theta)$.

27. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the equation whose ro $\alpha\beta + \beta\gamma$, $\beta\gamma + \gamma\alpha$, $\gamma\alpha + \alpha\beta$.

28. Solve by Cardan's method : $x^3 - 18x - 35 = 0$.

29. Find a parametric representation of the ellipsoid $x^2 + y^2 + \frac{1}{4}z^2 = 1$. also find a unit vector to the surface.

30. Find the real root of the equation $x \log_{10} x - 1.2 = 0$ correct to five decimal places by the method of false position.

31. Using Newton-Raphson method, find correct to four decimals the root between 0 and 1 of the equation $x^3 - 6x + 4 = 0$.

$(6 \times 4 = 24)$

Part D

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) Evaluate $\iint_C A \cdot \hat{n}\, dS$ where $A = yz\hat{i} + zx\hat{j} + xy\hat{k}$ and S is the region bounded by $x^2 + y^2 + z^2 = 1$, in the first octant.

(b) Find by Green's Theorem, the value of $\int_C \left(x^2 y\,dx + y\,dy\right)$ where C is the closed curve formed by $y^2 = x$ and $y = x$ between (0, 0) and (1, 1).

33. (a) Solve by Ferrari's method : $x^4 - 10x^3 + 35x^2 - 50x + 24 = 0$.

(b) Solve $x^3 - 9x^2 + 14x + 24 = 0$, two of whose roots being in the ratio 3 : 2

34. (a) State Gauss's Divergence Theorem. Use it to evaluate $\iint_S \bar{F} \cdot \bar{n}\, dS$ where

$\bar{F} = \left(x^2 - yz\right)\bar{i} + \left(y^2 - zx\right)\bar{j} + \left(z^2 - xy\right)\bar{k}$ over the rectangular parallelopiped

$0 \le x \le a, 0 \le y \le b, 0 \le z < c$.

(b) Prove that $\mathrm{div}\,\phi\mathbf{f} = \phi\,\mathrm{div}\,\mathbf{f} + \mathbf{f}.(\mathrm{grad}\,\phi)$ and hence show that $div\left(r^3\bar{r}\right) = 6r^3$.

35. (a) Find a real root of $x^4 - x - 10 = 0$ by Newton-Raphson method.

(b) Find the root of the equation $x^3 - x - 11 = 0$, using bisection method correct to three decimal places which lies between 2 and 3.

$(2 \times 15 = 30)$