**E 6899** (Pages : 4) Reg. No.................................

Name.....................................

## B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, NOVEMBER 2013

### First Semester

Complementary Course—OPERATIONS RESEARCH—LINEAR PROGRAMMING

(For B.Sc. Mathematics Vocational—Model II)

[2013 Admissions]

Time : Three Hours Maximum : 80 Marks

### Part A (Short Answer Questions)

*Answer* **all** *questions.*
*1 mark for each question.*

1. When a set of vectors $X_1, X_2, \ldots, X_n$ of a vector space V are said to be linearly independent ?
2. Define norm of a vector X.
3. When a square matrix A is said to be singular ?
4. Give an example of a bounded set S in En.
5. When a set $K \subseteq E_n$ is said to be convex ?
6. Find the inner product of the vectors $[2 -3\ 4]'$ and $[4\ \ 2\ \ -3]'$.
7. Find H (X) for $f(X) = x_1^3 + 2x_2^3 + 3x_1 x_2 x_3 + x_3^2$.
8. Find the unit vector in the direction of the steepest ascent of $f(X) = x_1^2 + 2x_1 x_2 + x_1 x_3 + x_2 x_4 + x_4^2$ at the point (1, 0, –1, 1).
9. Define feasible solution of a LP problem.
10. What are artificial variables ?

(10 × 1 = 10)

### Part B (Brief Answer Questions)

*Answer any* **eight** *questions.*
*Each question carries 2 marks.*

11. Prove that intersection of two convex sets is a convex set.
12. Prove that the set of vertices of a convex polyhedron is a subset of its spanning points.
13. Show that the vectors $[1 -2 -2]'$ and $[2 -1\ \ 2]'$ are orthogonal. Find a vector orthogonal to both these vectors.

**Turn over**

14. Show that $|X + Y| = |X| + |Y|$ if either $Y = \lambda X, \lambda \geq 0$ or $X = 0$ or $Y = 0$.

15. Define the convex hull of a set. Find the convex hull of the set $S = \{X \in E_n : 1 < |X| < 2\}$.

16. Write out in full the quadratic form whose matrix is $\begin{bmatrix} 2 & -3 & 1 \\ -3 & 4 & 2 \\ 1 & 2 & -6 \end{bmatrix}$.

17. Write Taylor series for $f(x) = x_1^2 + 3x_1 x_2 - 4x_2^2 + 4x_1 + 5x_2 x_3 - x_3^2$ about the origin.

18. Find the directional derivative of $f(X) = 2x_1^3 x_2 - 3x_2^2 x_3$ at the point (1, 2, –1) in a direction towards the point (3, –1, 5).

19. Prove that $f(x) = x^2, x \in \mathbb{R}$, is a convex function.

20. State the general LP problem.

21. What are basic solutions ?

22. What is degeneracy in a LP problem ?

(8 × 2 = 16)

## Part C (Short Essay Questions)

*Answer any* **six** *questions.*
*Each question carries 4 marks.*

23. Determine whether the vector $[6 \ 1 \ -6 \ 2]'$ is in the vector space generated by the vectors $[1 \ 1 \ -1 \ 1]'$, $[-1 \ 0 \ 1 \ 1]'$, $[1 \ -1 \ -1 \ 0]'$. What is the dimension of the vector space ?

24. Find a set of linearly independent solutions of

$$4x_1 - x_2 + 2x_3 + x_4 = 0,$$
$$2x_1 + 3x_2 - x_3 - 2x_4 = 0.$$

and then write a general solution.

25. Define the δ-neighbourhood of a point X in $E_n$. Also prove that the δ–neighbourhood of a point X in $E_n$ is a convex set.

26. Find the eigenvalues of the matrix of the quadratic form $2x_1^2 + 4x_1 x_2 + 2x_2^2 + x_3^2$ and determine the nature of the form.

27. Find the point in the plane $x_1 + 2x_2 + 3x_3 = 1$ in $E_3$ which is nearest to the point (–1, 0, 1).

28. Let $f(X)$ be defined in a convex domain $K \subseteq E_n$ and be differentiable. Then prove that $f(X)$ is a convex function if and only if $f(X_2) - f(X_1) \geq (X_2 - X_1)' \nabla f(X_1)$ for all $X_1$, $X_2$ in K.

29. Prove that a vertex of the set $S_F$ of feasible solutions of a LP problem is a basic feasible solution.

30. Solve graphically the following LP problem :

Maximize $2x_1 + 5x_2$

subject to
$$\begin{aligned} x_1 + x_2 &\leq 11 \\ 2x_1 + 5x_2 &\leq 40 \\ x_2 &\geq 4 \\ x_1 \geq 0, x_2 &\geq 0. \end{aligned}$$

31. Use simplex method to solve the following LP problem :

Minimize $x_1 - 3x_2 + 2x_3$

subject to
$$\begin{aligned} 3x_1 - x_2 + 2x_3 &\leq 7 \\ -2x_1 + 4x_2 &\leq 12 \\ -4x_1 + 3x_2 + 8x_3 &\leq 10 \\ x_1 \geq 0, x_2 \geq 0, x_3 &\geq 0. \end{aligned}$$

(6 × 4 = 24)

## Part D (Essay)

*Answer any* **two** *questions.*
*Each question carries* 15 *marks.*

32. (a) Prove that a square matrix Ar of rank r is non-singular if and only if its column vectors are linearly independent.

(b) Prove that any intersection of closed set is closed.

(c) Prove that every point of [S] can be expressed as a convex linear combination of atmost $n + 1$ points of $S \subseteq E_n$.

33. (a) Prove that a point $X_v$ of a polytope is a vertex if and only if $X_v$ is the only member of the intersection set of all the generating hyperplanes containing it.

(b) Find all the basic solutions of the following equations identifying in each case the basic vectors and the basic variables :

$$x_1 + x_2 + x_3 = 4$$
$$2x_1 + 5x_2 - 2x_3 = 3.$$

(c) Let K be a non-empty closed bounded convex set in $E_n$ and P a supporting hyperplane. Prove that $K \cap P$ is non-empty closed bonded convex set of dimension $(n - 1)$.

34. (a) Find the relative maxima and minima and saddle points, if any, of

$$f(X) = x_1^3 + x_2^3 - 3x_1 - 12x_2 + 25.$$

(b) Use the method of Lagrange multipliers to find the maxima and minima of $x_2^2 - (x_1 - 1)^2$

subject to $x_1^2 + x_2^2 \leq 1$.

(c) Prove that $f(X) = 2x_1^2 + 2x_2^2 + 4x_3^2 + 2x_1 x_2 + 2x_1 x_3 + 4x_2 x_3$ is a convex function.

35. Consider the LP problem :

Minimize $x_1 + 3x_2$

subject to
$$x_1 + x_2 \geq 3$$
$$-x_1 + x_2 \leq 2$$
$$x_1 - 2x_2 \leq 2$$
$$x_1 \geq 0, x_2 \geq 0.$$

(a) Solve it graphically.

(b) Solve it by using the Big-M method.

(c) Solve it by using the two-phase simplex method.

$(2 \times 15 = 30)$