E 2552 (Pages : 3) Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, NOVEMBER 2011

First Semester

LINEAR PROGRAMMING

[Complementary Course for B.Sc. Mathematics (Vocational Model II)]

Time : Three Hours Maximum Weight : 25

Part A (Objective Type Questions)

Answer **all** *questions.*
A bunch of **four** *questions carries weight* 1.

I. 1 Define a Vector Space.

2 Define linear independent vectors.

3 Show that $W = \{X | X = (x_1, 0, x_3, ..., x_n)\}$ is a subspace of R_n.

4 Verify whether the vectors $[1, -2, -2]'$ and $[2, -1, 2]'$ are orthogonal or not ?

II. 5 Explain how matrix notations can be used to represent a system of 'm' non–homogeneous linear equations in the 'n' unknowns $x_1, x_2, ..., x_n$.

6 Show that the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ is singular.

7 Are the equations

$x_1 + x_2 = 4, 2x_1 + x_2 = 6$ consistent ?

8 Define the term "The basic solution" for a system of linear equations.

III. 9 Show that the intersection of two convex sets is again a convex set.

10 Test the nature of the quadratic form

$3x_1^2 + 2x_1 x_2 + x_2^2$.

11 Explain the terms local extrema and global extrema.

12 How to define a saddle point ?

IV. 13 For a L.P. problem prove that the set S_F of feasible solutions, if not empty, is a closed convex set bounded from below having atleast one vertex.

14 Define slack and surplus variables used in Simplex method.

15 What are Simplex multipliers ?

16 What are the different steps in the revised Simplex method ?

(4 × 1 = 4)

Turn over

Part B (Short Answer Type Questions)

Answer any **five** *questions.*
Each question has weight 1.

17 Define a Euclidean space.

18 Define the term "Norm" of a vector.

19 Find the inner product of the vectors [2, –3, 4]' and [4, 2, –3]'.

20 Explain the case of obtaining non-trivial solutions for a homogeneous system of linear equations.

21 Show that the convex polyhedron is a convex set.

22 Write the quadratic form whose matrix is $\begin{bmatrix} 7 & -2 & 1 \\ -2 & 10 & -2 \\ 1 & -2 & 7 \end{bmatrix}$.

23 Write a short note on the Two Phase method for solving a L.P. problem.

24 What are the limitations of graphical method in solving a L.P. problem.

(5 × 1 = 5)

Part C (Short Essay Questions)

Answer any **four** *questions.*
Each question carries a weight of 2.

25 For any pair of n-vectors X, Y. Prove the Cauchy-Schwarz inequality in the form $|X'Y| \le |X||Y|$.

26 If A is any $r \times n$ matrix, $r \le n$, with linearly independent row vectors, then prove that there is atleast one $r \times r$ submatrix of A which is non–singular.

27 Define the convex hull [S] of a set $S \subset E_n$. Prove that every point of [S] can be expressed as a Convex linear combination of at most $(n + 1)$ points of S.

28 Find the points in the plane $x_1 + 2x_2 + 3x_3 = 1$ in E_3 which is nearest to the point (–1, 0, 1). Also find the minimum distance.

29 Use graphical method to solve the L.P. Problem.

Maximise $Z = 6x_1 + 4x_2$

subject to

$$2x_1 + 4x_2 \le 4$$
$$4x_1 + 8x_2 \ge 16$$

and $x_1, x_2 \ge 0$.

30 Describe degeneracy in an Optimisation Problem.

(4 × 2 = 8)

Part D (Essay Questions)

Answer any **two** *questions.*
Each question carries a weight of 4.

31 Find the maximum and minimum values of $|X|^2$, $X \in E_3$ subject to the constraints

$$g_1(X) = \frac{x_1^2}{4} + \frac{x_2^2}{5} + \frac{x_3^2}{25} - 1 = 0$$

$$g_2(X) = x_1 + x_2 - x_3 = 0.$$

32 If the set of feasible solutions S_F is non–empty, prove that the objective function $f(X)$ has either an unbounded minimum or it is a minimum at a vertex of S_F.

33 Use simplex method to solve the L.P. problem

Maximise $Z = 4x_1 + 10x_2$

subject to

$$2x_1 + x_2 \le 50$$
$$2x_1 + 5x_2 \le 100$$
$$2x_1 + 3x_2 \le 90$$

and $x_1, x_2 \ge 0$.

(2 × 4 = 8)