E 2488 (Pages : 4) Reg. No.................................

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B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, NOVEMBER 2015

First Semester

Complementary Course—OPERATIONS RESEARCH—LINEAR PROGRAMMING

(For B.Sc. Mathematics Vocational—Model II)

[2013—Admission onwards]

Time : Three Hours Maximum : 80 Marks

Part A (Short Answer Questions)

Answer **all** *questions. 1 mark for each question.*

1. Define norm of a vector X in E_n.
2. Find a vector orthogonal to both the vectors $X_1 = [1, -2, -2]^1$ and $X_2 = [2, -1, 2]^1$.
3. Find the norm of the vector [2, –1, 4, –3].
4. Check the consistency of the system of equations

$$x_1 + x_2 = 4$$
$$2x_1 + x_2 = 6$$

5. When will you say that a system of equations AX = B is homogeneous ?
6. Give an example of a 2 × 2 singular matrix.
7. Define vertices of a convex set K in E_n.
8. What do you mean by a convex function ?
9. Define a basic solution of a Linear programming problem.
10. Explain the term Artificial variables in a linear programming problem.

(10 × 1 = 10)

Part B (Brief Answer Questions)

Answer any **eight** *questions.*
Each question carries 2 marks.

11. State and prove the triangle inequality for Eucledian norm.
12. Prove that if a vector X is orthogonal to every vector of a basis of a Eucledian space, then X is orthogonal to every vector in that space.
13. Define the convex hull of a set. Find the convex hull of the set $A = \{X \in En / |x| \geq 1\}$.
14. Consider the system of equation : AX = B discuss about solution of this system interms of nature of A.

Turn over

15. What do you mean by a convex linear combination ? Give an example in E_3.

16. Prove that intersection of two convex set is convex.

17. Write down the Taylor series for the function $f(x) = x_1^2 + 5x_1x_2 - 3x_2^2 + x_3$ about the point (1, 0, 0).

18. Prove that $d(X) = CX, X \in E_n$ is both convex and concave.

19. Write down the standard form of a linear programming problem.

20. Determine whether the vector $[6, 1, -6, 2]^1$ is in the vectorspace generated by the vectors $[1, 1, -1, 1]^1$, $[-1, 0, 1, 1]^1$, $[1, -1, -1, 0]^1$. Find dimension of the vector space.

21. What are the limitations of graphical method in solving a linear programming problem ?

22. Find H(X) for $f(X) = x_1^2 + 4x_2^2 + 4x_3^2 + 4x_1x_2 + 4x_1x_3 + 16x_2x_3$.

(8 × 2 = 16)

Part C (Short Essay Questions)

Answer any **six** *questions.*
Each question carries 4 marks.

23. If $X \in E_n$ and $V \subseteq E_n$ such that $V = \{X / X = [x_1, x_2, ...x_n]^1, x_1 + x_2 + ... + x_n = 0\}$ then check whether V is a subspace of E_n or not, if so, give a geometric interpretation for $n = 3$.

24. Determine those values of λ for which the following system of equations have a non-trivial solution

$$3x_1 + x_2 - \lambda x_3 = 0$$
$$4x_1 - 2x_2 - 3x_3 = 0$$
$$2\lambda x_1 + 4x_2 + \lambda x_3 = 0.$$

For each value of λ, find the general solution.

25. For each pair of n-vectors X ; Y state and prove the Cauchy-Schwarz inequality also explain it in E_2.

26. If A is an $r \times n$ matrix, $r \leq n$ with linearly independent row vectors, then there is at least one $r \times r$ submatrix of A which non singular.

27. Prove that every point of [δ] can be expessed as a convex linear combination of atmost $n + 1$ points of $\delta \subseteq E_n$.

28. Find the point in the plane : $x_1 + 2x_2 + 3x_3 = 1$ in E_3 which is nearest to the point (–1, 0, 1).

29. Solve graphically the following linear programming problem :

Maximize $5x_1 + 3x_2$

Subject to $4x_1 + 5x_2 \le 10$

$5x_1 + 2x_2 \le 10$

$3x_1 + 8x_2 \le 12$

$x_1, x_2 \ge 0.$

30. Solve the following linear programming problem using simplex method :

Maximize $5x_1 + 3x_2 + x_3$

Subject to $2x_1 + x_2 + x_3 = 3$;

$-x_1 + 2x_3 = 4$

$x_1, x_2, x_3 \ge 0.$

31. Construct a set of three mutually orthogonal unit vectors which are linear combination of the vectors.

$X_1 = [1, 0, 2, 2]^1$; $X_2 = [1, 1, 0, 1]^1$ and $X_3 = [1, 1, 0, 0]^1$.

(6 × 4 = 24)

Part D (Essays)

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) Let X, Y $\in E_n$, subject to $|X| = (x_1^2 + x_2^2 + ... + x_n^2)^{1/2}$ is a norm and also

subject to $|X + Y|^2 + |X - Y|^2 = 2|X|^2 + 2|Y|^2$ and give a geometric interpretation in E_2.

(b) Let V be a set of all polynomials in x of degree n or less and define the sum and product as $(f + g)(x) = f(x) + g(x)$; $(cf)(x) = c\,f(x)$. For all $f, g \in V$ and c any real number then prove that V is a vertor space.

33. (a) Find a set of linearly independent solutions of $4x_1 - x_2 + 2x_3 + x_4 = 0, 2x_1 + 3x_2 - x_3 - 2x_4 = 0$. and then write a general solution.

(b) Find all the basic solutions of the following equations, identity in each case, the basic vectors and the basic variables $x_1 + x_2 + x_3 = 4$; $2x_1 + 5x_2 - 2x_3 = 3$.

Turn over

(c) Let X_0 be a particular solution of the system of equations $AX = B$ and let Y be a linear combination of the linearly independent solutions of $AX = 0$. Show that $X_0 + Y$ is the general solution of $Ax = B$.

34. (a) A point X_v of a polytop is a vertex if and only if X_v is the only member of the intersection set of all the generating hyperplanes containing it.

(b) Let K be a non-empty closed bounded convex set in E_n and P a supporting hyperplane. Prove that $K \cap P$ is nonempty closed bounded convex set of dimension $(n-1)$.

(c) Use the method of Lagrange multipliers to find the minima and maxima of $(x_1 - 4)^2 + (x_2 - 3)^2$ Subject to $36(x_1 - 2)^2 + (x_2 - 3)^2 = 9$.

35. (a) Solve the following linear programming problem by simplex method :

$$\begin{aligned} &\text{Maximize } 5x_1 + 3x_2 + x_3 \\ &\text{Subject to } 2x_1 + x_2 + x_3 = 3 \\ &\qquad -x_1 + 2x_3 = 4 \\ &\qquad x_1, x_2, x_3 \geq 0. \end{aligned}$$

(b) Consider the linear programming problem :

$$\begin{aligned} &\text{Maximize } 5x_1 - x_2 \\ &\text{Subject to } x_1 + x_2 \geq 2 \\ &\qquad x_1 + 2x_2 \leq 2 \\ &\qquad 2x_1 + x_2 \leq 2 \\ &\qquad x_1 \geq 0, x_2 \geq 0 \end{aligned}$$

(i) Solve by using the Big-M method.

(ii) Solve by using the two phase simplex method.

(2 × 15 = 30)