E 9756 (Pages : 4) Reg. No..............................

Name..................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, NOVEMBER 2014

First Semester

Complementary Component : OPERATIONS RESEARCH—LINEAR PROGRAMMING

(For B.Sc. Mathematics Vocational–Model II)

[2013 Admission onwards]

Time : Three Hours Maximum : 80 Marks

Part A

Short Answer Questions.
Answer **all** *questions.*
1 *mark for each question.*

1. When a subset W of a vector space V is said to be a subspace of V ?
2. Show that the vectors $\left[1, {}^{-}2, {}^{-}2\right]'$ and $\left[2, {}^{-}1, 2\right]'$ are orthogonal.
3. Define the Euclidean norm of an n-vector X.
4. When a set $S \subseteq E_n$ is said to be closed ?
5. Find the convex hull of the set $S = \{X \in E_n : 1 < |X| < 2\}$.
6. Write the quadratic form $x_1^2 - 2x_2^2 - 4x_3^2 + 4x_1 x_2 + 6x_1 x_3 - 8 x_2 x_3$ in the form X'AX.
7. When the function $f(Y, Z)$ is said to have a saddle point at (Y_0, Z_0)?
8. Find $\nabla f(X)$ if $f(X) = x_1^3 + 2x_2^3 + 3x_1 x_2 x_3 + x_3^2$.
9. Define feasible solution of a LP problem.
10. What are slack variables ?

(10 × 1 = 10)

Part B

Brief Answer Questions.
Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. Show that if $X \in E_n$ and $V \subset E_n$ such that

$$V = \left\{X : X = [x_1, x_2, \ldots, x_n]', \; x_1 + x_2 + \ldots + x_n = 0\right\}, \text{ then V is a subspace of } E_n.$$

Turn over

12. For any pair of n-vectors X, Y prove that $|X' Y| \le |X||Y|$.

13. Prove that the convex hull of a set S is the set of all convex linear combinations of points in S.

14. Prove that all internal points of a convex set K themselves constitute a convex set.

15. Write out in full the quadratic form whose matrix is $\begin{bmatrix} 1 & 2 & 4 \\ 2 & 6 & -2 \\ 4 & -2 & 14 \end{bmatrix}$.

16. Write Taylor series for $f(x) = x_1^2 + 3x_1 x_2 - 4x_2^2 + 4x_1 + 5x_2 x_3 - x_3^2$ about the point (1, 1, 1).

17. Examine $f(X) = x_1^2 + 4x_2^2 + 4x_3^2 + 4x_1 x_2 + 4x_1 x_3 + 16 x_2 x_3$ for relative extrema.

18. Let $X \in E_n$ and let $f(X) = X' A X$ be a quadratic form. if $f(X)$ is positive semidefinite, then $f(x)$ is a convex function.

19. State the general LP problem.

20. Prove that the set S_F of feasible solutions of a LP problem is a convex set.

21. Find all basic solutions of the following system of equations :

$$\begin{aligned} 2x_1 - x_2 + 3x_3 &= 3 \\ x_1 - 2x_2 \quad\quad &= 4. \end{aligned}$$

22. What is degeneracy in a LP problem ?

(8 × 2 = 16)

Part C

Descriptive / Short Essay Type Questions.
Answer any **six** *questions.*
Each question carries 4 *marks.*

23. If A is an $r \times n$ matrix, $r \le n$, with linearly independent row vectors, then prove that there is at least on $r \times r$ submatrix of A which is non-singular.

24. Test the following equations for consistency :

$$\begin{aligned} x_1 + x_2 + 2x_3 + x_4 &= 5 \\ 2x_1 - 3x_2 - x_3 - 2x_4 &= 2 \\ 4x_1 + 5x_2 + 3x_3 \quad\quad &= 7. \end{aligned}$$

25. Let K be a closed convex set and X_0 be a point not in K. Then prove that there exists a hyperplane which contains X_0 such that K is contained in one of the half spaces produced by the hyerplane.

26. Find the point in the plane $x_1 + 2x_2 + 3x_3 = 1$ in E_3 which is nearest to the point (–1, 0, 1).

27. Let $f(x)$ be a convex differentiable function defined in a convex domain $K \subseteq E_n$. Then prove that $f(x)$, $X_0 \in K$ is a global minimum if and only if $(X - X_0)' \ \nabla f(X_0) \geq 0$ for all X in K.

28. Find a point Y in E_4 such that $|Y - X_o| = 4$ and $Y - X_o$ is the vector of steepest desent for the function $f(X) = x_1^2 - 3x_1 x_2 + 4x_1 x_3 + x_3 + 4x_4^2$ at the point $X_o = (1, 0, -1, -1)$.

29. Prove that a vertex of the set S_F of feasible solutions of a LP problem is a basic feasible solution.

30. Solve graptically the following LP problem :

Maximize $5x_1 + 3x_2$

Subject to
$$4x_1 + 5x_2 \leq 10$$
$$5x_1 + 2x_2 \leq 10$$
$$3x_1 + 8x_2 \leq 12$$
$$x_1 \geq 0, x_2 \geq 0.$$

31. Solve the following problem using simplex method :

Maximize $5x_1 + 3x_2 + x_3$

Subject to
$$3x_1 - x_2 + 2x_3 \leq 7$$
$$-2x_1 + 4x_2 \leq 12$$
$$-4x_1 + 3x_2 + 8x_3 \leq 10$$
$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$$

$(6 \times 4 = 24)$

Part D

Long Essay Type Questions.
Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) Construct a set of three mutually orthogonal unit vectors which are linear combinations of the vectors $X_1 = [1\ 0\ 2\ 2]'$, $X_2 = [1\ 1\ 0\ 1]'$, $X3 = [1\ 1\ 0\ 0]'$

(b) Prove that any intersection of closed sets is closed.

(c) Prove that for a set K to be convex it is necessary and sufficient that every convex linear combination of points in K belongs to K.

33. (a) Define a convex polyhedron. Prove that every convex polyhedron is a convex set.

Turn over

(b) Find a set K is non-empty, closed, convex and bounded from below (or above), then prove that it has at least one vertex.

(c) Find the eigenvalues of the matrix of the quadratic form $2x_1^2 + 4x_1 x_2 + 2x_2^2 + x_3^2$ and determine the nature of the form.

34. (a) Find the point on the surface $z = x^2 + y^2$ which is nearest to the point (3, −6, 4).

(b) Let $f(x)$ be defined in a convex domain $K \subseteq E_n$ and be differentiable. Then prove that $f(x)$ is a convex function if and only if $f(x_2) - f(x_1) \geq (x_2 - x_1)' \nabla f(x_1)$ for all x_1, x_2 in K.

(c) Use the method of Lagrange multipliers to find the maxima and minima of

$(x_1 - 4)^2 + (x_2 - 3)^2$ subject to $36(x_1 - 2)^2 + (x_2 - 3)^2 = 9$.

35. Consider the LP problem ;

Maximize $5x_1 - x_2$

Subject to
$$x_1 - x_2 \geq 2$$
$$x_1 + 2x_2 \leq 2$$
$$2x_1 + x_2 \leq 2$$
$$x_1 \geq 0, x_2 \geq 0.$$

(a) Sove it graphically.

(b) Solve it by using the Big-M method.

(c) Solve it by using two-phase simplex method.

(2 × 15 = 30)