E 4267 (Pages : 4) Reg. No.................................

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B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, NOVEMBER 2016

First Semester

Complementary Course—OPERATIONS RESEARCH—LINEAR PROGRAMMING

(For B.Sc. Mathematics Vocational—Model II)

[2013 Admission onwards]

Time : Three Hours

Maximum Marks : 80

Part A (Short Answer Questions)

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Define a Euclidean norm of an n vector X.
2. Find a vector orthogonal to both the vectors $[1 \ \ -2 \ \ -2]^1$ and $[2 \ \ -1 \ \ 2]^1$.
3. Give an example of two linearly independent vectors in a vector space.
4. Test the consistency of the system of equations $x_1 + x_2 = 5,\ 2x_1 + 3x_2 = 2$.
5. Give an example of a 2×2 singular matrix.
6. Define a boundary point of a set.
7. What do you mean by a convex function ?
8. Let $S = \{X \in E_n / |x| \geq 1\}$ then find the convex hull of S.
9. Define the term optimal solution of a L.P. problem.
10. Find any one of the basic solution of the system

$$\begin{aligned} 2x_1 - x_2 + 3x_3 &= 3 \\ x_1 + 2x_2 - x_3 &= 4. \end{aligned}$$

$(10 \times 1 = 10)$

Part B (Brief Answer Questions)

Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. Will [2 0 0], [3 3 0]' and [1 1 1] form a basis for R_3. Give your arguments in detail.
12. Prove that if a vector X is orthogonal to every vector of a basis of a Euclidean space then X is orthogonal to every vector in that space.
13. Prove that $W = \{X / X = (0, 0, x_3, \ldots x_n)\}$ is a subspace of R_n.

Turn over

14. Find a set of linearly independent solution of

$$\begin{aligned} 4x_1 - x_2 - 2x_3 + x_4 &= 0 \\ 2x_1 + 3x_2 - x_3 - 2x_4 &= 0 \end{aligned}$$

and then write a general solution.

15. Consider the system of equations AX = B discuss about the solution of this system of equations in terms of nature of the matrix A.

16. State the implicit function theorem.

17. Prove that intersection of two convex sets is a convex set.

18. Indicate the expression $4x_1x_2 - x_1^2 - 4x_2^2 - x_3^2$ is positive or negative definite or indefinite.

19. Write the quadratic form $x_1^2 - 2x_2^2 - 4x_3^2 + 4x_1x_2 + 6x_1x_3 - 8x_2x_3$ in X'AX form.

20. Write the following L.P. Problem in the canonical form :

Minimize $f = 2x_1 + x_2 - x_3$

$$\begin{aligned} \text{subject to } 2x_1 - 5x_2 + 3x_3 &\le 4 \\ 3x_1 + 6x_2 - x_3 &\ge 2 \\ x_1 + x_2 + x_3 &= 4 \end{aligned}$$

$x_1 \ge 0, x_3 \ge 0, x_2$ unrestricted.

21. What do you mean by an artificial variable in a L.P. problem ?

22. Solve graphically Maximize $4x_1 + 5x_2$

subject to $x_1 - 2x_2 \le 2, 2x_1 + x_2 \le 6, x_1 + 2x_2 \le 5, -x_1 + x_2 \le 2, x_1 + x_2 \ge 1, x_1, x_2 \ge 0.$

(8 × 2 = 16)

Part C (Short Essay Type Questions)

Answer any **six** *questions.*
Each question carries 4 *marks.*

23. Construct a set of three mutually orthogonal unit vectors which are linear combinations of the vectors $X_1 = [1\ 0\ 2\ 2]'$ $X_2 = [1\ 1\ 0\ 1]'$ $X_3 = [1\ 1\ 0\ 0]'$.

24. If $X \in E_n$ and $V \subseteq E_n$ such that $V = \left\{X/X = [x_1, x_2, \ldots x_n]', x_1 + x_2 + \ldots + x_n = 0\right\}$ then check whether V is a subspace of E_n or not, if so, give a geometric interpretation for $n = 3$.

25. Determine those values of λ for which the following equation have a non-trivial solution

$$\begin{aligned} 3x_1 + x_2 - \lambda x_3 &= 0 \\ 4x_1 - 2x_2 - 3x_3 &= 0 \\ 2\lambda x_1 + 4x_2 + \lambda x_3 &= 0 \end{aligned}$$

for each value of λ, find the general solution.

26. Are the following system of equation is constant ? Justify. What is the rank of the matrix A $x_1 + x_2 + x_3 + x_4 = 0, x_1 + 3x_2 + 2x_3 + 4x_4 = 0, 2x_1 + x_3 - x_4 = 0.$

27. Find the directional derivative of $f(x) = 2x_1^2x_2 - 3x_2^2x_3$ at the point $X_0 = (1, 2, -1)$, in the direction towards the point $Y = (3, -1, 5)$. Find also the maximum directional derivative at X_0.

28. Let $X \in E_n$ and let $f(X) = X'AX$ be a quadratic form. If $f(X)$ is positive semidefinite then prove that $f(X)$ is a convex function.

29. Define the relative maxima and minima of a function. Also find the relative maxima and minima and saddle point if any of $f(X) = x_1^3 + x_2^3 - 3x_1 - 12x_2 + 25$.

30. Solve the following L.P. Problem by revised simplex method :

Maximize $-5x_1 + x_2 - x_3 + 10x_4 - 7x_5$

subject to
$$\begin{aligned} 3x_1 - x_2 - x_3 + x_4 &= 4 \\ x_1 - x_2 + x_3 + x_4 &= 1 \\ 2x_1 + x_2 + x_4 + x_5 &= 7 \end{aligned}$$
$x_1, x_2, x_3, x_4, x_5 \geq 0$.

31. Solve the L.P. Problem by Big-M method :

Maximize $Z = 2x_1 - 3x_2 - 2x_3$

subject to
$$\begin{aligned} 2x_1 + x_2 - 2x_3 &\leq 3 \\ x_1 + x_2 - x_3 &\geq 1 \end{aligned}$$
$x_1, x_2, x_3 \geq 0$.

$(6 \times 4 = 24)$

Part D (Long Essay Type Questions)

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) Let V be a set of all polynomials in x of degree n or less than n and define the sum and product as $(f + g)x = f(x) + g(x)$ and $(cf)(x) = cf(x)$ for all $f, g \in V$ and C any real number then prove that V is a vector space.

(b) Let $X, Y \in E_n$ S.T. $|X| = \left(x_1^2 + x_2^2 + \ldots + x_n^2\right)^{1/2}$ satisfies the definition of theorem.

(c) Show that $|X + Y|^2 + |X - Y|^2 = 2|X|^2 + 2|Y|^2$.

33. (a) Solve the equations $x_1 + x_2 - 2x_3 + x_4 + 3x_5 = 1$, $2x_1 - x_2 + 2x_3 + 2x_4 - 6x_5 = 2$, $3x_1 + 2x_2 - 4x_3 - 3x_4 - 9x_5 = 3$.

(b) Obtain all the basic solutions and why x_1 is always in the basis.

(c) Find all the basic solution of the following equations. Identify in each case the basis vectors and basic variables :

$$\begin{aligned} x_1 + x_2 + x_3 &= 4 \\ 2x_1 + 5x_2 - 2x_3 &= 3. \end{aligned}$$

Turn over

34. (a) Prove that the convex polyhedron is a convex set and the set of vertices of a convex polyhedron is a subset of its spanning points.

(b) Use the method of Lagrange multipliers to find the maxima and minima of $(x_1-4)^2+(x_2-3)^2$ subject to $36(x_1-2)^2+(x_2-3)^2=9$.

35. (a) Explain briefly the simplex method.

(b) Consider the LP Problem :

Maximize $5x_1 - 3x_2 + 4x_3$

subject to $x_1 - x_2 < 1, -3x_1 + 2x_2 + 2x_3 \leq 1, 4x_1 - x_3 = 1, x_2 \geq 0$ $x_3 \geq 0$, x_1 unrestricted.

Write the problem in the standard form.

(c) Solve the L.P. Problem in (b) using Simplex method.

$(2 \times 15 = 30)$