B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, NOVEMBER 2013

First Semester

Core Course—FOUNDATION OF MATHEMATICS

(Common for Model I and Model II B.Sc. Mathematics and B.Sc. Computer Applications)

[2013 Admissions]

Time : Three Hours Maximum : 80 marks

Part A (Short Answer Questions)

Answer **all** *questions.*

Each question carries 1 *mark.*

1. Find the power set of the set $\{\phi, \{\phi\}\}$.
2. What are the terms a_0, a_1, a_2 and a_3 of the sequence $\{a_n\}$ where $a_n = 6\left(\frac{1}{3}\right)^n$?
3. What is a reflexive relation ?
4. What are the equivalence classes of the relation congruence modulo 2 ?
5. Define a lattice.
6. Write the negation of "This is a boring course".
7. Define a tautology.
8. State the fundamental theorem of arithmetic.
9. Find the remainder , when 8^{30} is divided by 31.
10. Find $\phi(200)$, where ϕ is the Euler's function.

(10 × 1 = 10)

Part B (Brief Answer Questions)

Answer any **eight** *questions.*

Each question carries 2 *marks.*

11. If $A = \{a, b, c, d\}$ and $B = \{y, z\}$. Find $A \times B$ and $B \times A$.
12. Determine whether the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x + 1$ is a bijection.

Turn over

13. Find the value of $\sum_{i=1}^{4} \sum_{j=1}^{3} ij$.

14. List the relations on {0, 1} that contains the pair (0, 1).

15. Give an example of a relation on the set of positive integers which is not symmetric but transitive. Justify your example.

16. Draw the Hasse diagram for the partial ordering $\{(A, B) \mid A \le B\}$ on the power set P (S), where S = {a, b, c}.

17. Show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent.

18. Find the truth value of $\forall x\,(x^2 \ge x)$ if the domain consists of (i) all real numbers ; (ii) all integers.

19. Use a direct proof to show that the sum of two odd integers is even.

20. Find the sum of divisors of 540.

21. Solve $3x \equiv 5 \pmod{11}$.

22. If $2^n + 1$ is a prime, prove that n is a power of 2.

(8 × 2 = 16)

Part C (Short Essay Type Questions)

Answer any **six** *questions.*

Each question carries 4 marks.

23. For any two sets A and B, prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$.

24. Show that the function $f(x) = ax + b$ from $\mathbb{R}$ to $\mathbb{R}$ is invertible, where a and b are constants with $a \neq 0$, and find the inverse of f.

25. Find the number of reflexive relations on a set with n elements.

26. Let S = { 1, 2, 3, 4, 5, 6}. List the ordered pairs in the equivalence relation R determined by the partition A_1 = {1, 2, 3}, A_2 = {4, 5} and A_3 = {6}.

27. Express the statement $\lim_{x \to a} f(x) = L$ using quantifiers.

28. Use logical equivalences to show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

29. Prove that every square number is one of the form $5n$, $5n \pm 1$.

30. If $n \geq 2$ prove that the sum of integers less than n and prime to n is $\frac{1}{2} n \phi(n)$.

31. Show that $\angle 18 + 1$ is divisible by 437.

$(6 \times 4 = 24)$

Part D (Essay)

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) Define the floor and ceiling functions and display the graphs of these functions.

(b) Show that the set of odd positive integers is a countable set.

(c) Prove that if x is a real number, then $[2x] = [x] + \left[x + \frac{1}{2}\right]$.

33. (a) Let $m > 1$ be a positive integer. Prove that the relation congruence modulo m is an equivalence relation on the set of integers.

(b) Explain how to use a zero-one matrix to represent a relation on a finite set. Suppose that the relation R on a set is represented by the matrix $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$.

Is R reflexive, symmetric, and/or antisymmetric ?

(c) Define (i) partial ordering ; (ii) total ordering. Show that the divisibility relation on the set of positive integers is a partial order but not a total order.

34. (a) Explain proof by contradiction and proof by contra position.

(b) Prove by contradiction that " if $3n + 2$ is odd then n is odd".

(c) Give a direct to proof to show that the product of two perfect squares is a perfect square.

35. (a) If a and b are any *two* numbers, prove that there exists a unique number of such that common divisors of a and b are the same as the divisors of g.

(b) State and prove Euler's extension of Fermat's theorem.

(c) Show that the ninth power of any number is one of the forms $19m$, $19m \pm 1$.

$(2 \times 15 = 30)$