E 9709 (Pages : 4) Reg. No..............................

Name..............................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, NOVEMBER 2014

First Semester

Core Course—FOUNDATION OF MATHEMATICS

[Common for Model I and Model II B.Sc. Mathematics and B.Sc. Computer Applications]

[2013 Admission onwards]

Time : Three Hours Maximum : 80 Marks

Part A (Short Answer Questions)

Answer **all** *questions.*
Each question carries 1 mark.

1. Find the cardinality of the set $\{a, \{a\}, \{a, \{a\}\}\}$.
2. Define a sequence.
3. What is a transitive relation ?
4. Represent the relation $\{(1,1), (1,2), (1,3)\}$ on $\{1,2,3\}$ with a matrix.
5. Define a well-ordered set.
6. Write the converse of "If the home team wins, then it is raining".
7. Define a counter example.
8. Define the greatest common division of *two* numbers.
9. Find the number of divisons of 300.
10. State Fermat's theorem.

$(10 \times 1 = 10)$

Part B (Brief Answer Questions)

Answer any **eight** *questions.*
Each question carries 2 marks.

11. Let $f, g : \mathrm{R} \to \mathrm{R}$ be defined by $f(x) = 2x + 3$ and $g(x) = 3x + 2$. Find the compositions $f \circ g$ and $g \circ f$.
12. Let g be a function from A to B and f be a function from B to C. If both f and g are one to one function show that the composition fog is one to one.

Turn over

13. Compute $\sum_{i=1}^{2} \sum_{j=1}^{3} (i+j)$.

14. Find the number of relations from a set with m elements to a set with n elements.

15. What are the sets in the partition of the integers arising from congruence modulo 4 ?

16. Show that the inclusion relation $\subseteq$ is a partial ordering on the power set of a set S.

17. Show that $\neg(p \wedge q)$ and $\neg p \vee \neg q$ are logically equivalent.

18. What are the negations of the statements $\forall x\left(x^2 > x\right)$ and $\exists x\left(x^2 = 2\right)$?

19. Prove or disprove the product of two irrational numbers is irrational.

20. Find the remainder when 2^{24} is divided by 17.

21. Evaluate $\phi(450)$, where ϕ is the Euler function.

22. Solve $5x \equiv 2 \pmod 7$.

$(8 \times 2 = 16)$

Part C (Descriptive/Short Essay Type Questions)

Answer any **six** *questions.*
Each question carries 4 *marks.*

23. For any *two* sets prove that $\overline{A \cup B} = \bar{A} \cap \bar{B}$.

24. Let f be a function from A to B. Let S and T be subsets of A. Show that $f(S \cup T) = f(S) \cup f(T)$.

25. Let $m > 1$ be a positive integer. Show that the relation congruence modulo m is an equivalence relation on the set of integers.

26. Determine whether $(P(S), \subseteq)$ is a lattice where $S = \{a, b, c\}$.

27. Use logical equivalences to show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent.

28. Prove by contradiction that "if $3n + 2$ is odd then n is odd.

29. Prove that the sequence of primes is end less.

30. Define Euler's function $\phi(n)$. If p is a prime and r is a positive integer, show that $\phi\left(p^r\right) = p^r\left(1 - \frac{1}{p}\right)$.

31. Show that the fifth power of any number N has the same right-hand digit as N.

(6 × 4 = 24)

Part D (Long Essay Type Questions)

Answer any **two** *questions.*

Each question carries 15 marks.

32. (a) If A, B, C are sets, using a membership table show that $\overline{A \cap B \cap C} = \bar{A} \cup \bar{B} \cup \bar{C}$.

(b) Prove that if x is a real number, then $[2x] = [x] + \left[x + \frac{1}{2}\right]$.

(c) Show that the set of positive rational numbers is countable.

33. (a) Prove that a relation R on a set A is transitive if and only if $R^n \subseteq R$ for $n = 1, 2, 3, ...$

(b) Let R be an equivalence relation on a set A. Prove that the statements for elements a and b of A are equivalent :

(i) $a \, R \, b$. (ii) $[a] = [b]$. (iii) $[a] \cap [b] \neq \phi$.

(c) Define a maximal element and the greatest element of a poset. Find the maximal elements of the poset $(\{2, 4, 5, 10, 12, 20, 25\}, \mid)$.

34. (a) Explain proof by contraposition and proof by contradiction.

(b) Prove that $\sqrt{2}$ is not rational using contradiction method.

(c) Give a direct to proof to show that the sum of two rational numbers is rational.

35. (a) Prove that every composite number N has at least one prime divisor and deduce that N can be expressed as a product of prime factors.

(b) If $d_1 = 1, d_2, \ldots, d_r = n$ are the divisors of a number n, prove that :

$$\phi(d_1) + \phi(d_2) + \ldots\ldots + \phi(d_r) = n.$$

(c) If p is a prime number prove that $\lfloor p-1 + 1$ is divisible by p.

(2 × 15 = 30)