**E 2499** (Pages : 3) Reg. No..............................

Name..............................

## B.Sc. DEGREE (C.B.C.S.S) EXAMINATION, NOVEMBER 2011

### First Semester

Core Course—FOUNDATION OF MATHEMATICS

(Common for B.Sc. Model–I, and Model–II B.Sc. Mathematics and B.Sc. Computer Applications)

Time : Three Hours Maximum Weight : 25

### Part A (Objective Type Questions)

*Answer* **all** *questions.*

*A bunch of four questions has weight* 1.

I. 1 Let A, B, C be three sets. Show that $\overline{A \cup (B \cap C)} = \left(\overline{C} \cup \overline{B}\right) \cap \overline{A}$.

2 Let $f$ and $g$ be a functions from the set of integers defined by $f(x) = 2x + 3$ and $g(x) = 3x + 2$. What is the composition of $g$ and $f$.

3. Find $\sum_{K=50}^{100} K^2$

4. Suppose that A = {1, 2, 3} and B = { 1, 2} Let R be the relation from A to B containing $(a, b)$ if $a \in A$, $b \in B$ and $a > b$. Write down the matrix representing R if $a_1 = 1$, $a_2 = 2$, $a_3 = 3$, $b_1 = 1$ and $b_2 = 2$.

II. 5. Let R = {(1, 1), (2, 1), (3, 2), (4, 3 )} Find the powers $R^n$, $n = 2, 3, 4 \ldots\ldots$

6 Consider the relation :

R = {(1, 1) (1, 3), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (4, 1)}

on the set { 1, 2, 3, 4}. Draw the directed graph of R.

7 Let R be the relation on the set of integers such that $a$R$b$ if and only if $a = b$ or $a = -b$. Show that R is an equivalence relation.

8 Define a well ordered set.

III. 9 Translate the following English sentence into a logical expression. "You can not ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old".

10 Show that the propositions $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent.

11 What is the negation of the statement $\exists x \left(x^2 = 2\right)$ ?

12 State which rule of inference is the basis of the argument : "It is below freezing now. Therefore, it is either below freezing or raining now".

Turn over

IV. 13 Show that $2^n + 1$ is divisible by 3 if $n$ is odd.

14 Find $x$ such that $17x \equiv$ (mod 43).

15 If $p$ is an odd prime number and $a$ is prime to $p$, then show that $a^{\frac{1}{2}(p-1)} \equiv \pm 1 \pmod{p}$

16 Find the number of divisors of 7128.

(4 × 1 = 4)

## Part B (Short Answer Questions)

*Answer any* **five** *questions.*
*Each question has weight 1.*

17 Let A and B be subsets of a universal set U. Show that $A \subseteq B$ if and only if $\overline{B} \subseteq \overline{A}$.

18 Suppose that the relations $R_1$ and $R_2$ on a set A are represented by the matrices

$$M_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \text{ and } M_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

Write the matrices representing $R_1 \cup R_2$ and $R_1 \cap R_2$.

19 What are the sets in the partition of the integers arising from congruence modulo 4.

20 Construct a truth table for the compound proposition $(p \vee q) \rightarrow (p \oplus q)$.

21 Express the definition of a limit using quantifiers.

22 Prove that the product of any $n$ consecutive integers is divisible by $\underline{|n}$.

23 Define $\phi(n)$? If $p$ is a prime, then show that $\phi(p^r) = p^r\left(1 - \frac{1}{p}\right)$

24 Prove that 2, 4, 6 are roots of $5x^3 + 3x^2 - 4x - 2 \equiv 0 \pmod{7}$

(5 × 1 = 5)

## Part C (Short Essay Questions)

*Answer any* **four** *questions.*
*Each question has weight 2.*

25 If $x$ is a real number then prove that $[2x] = [x] + \left[x + \frac{1}{2}\right]$

26 Prove that the relation R on a set A is transitive if and only if $R^n \subseteq R$ for $n = 1, 2, 3, \ldots$

27 Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

28 Prove that $\sqrt{2}$ is irrational by giving a proof by contradiction.

29 Prove that a composite number can be expressed as the product of prime factors in one and only one way.

30 State and prove that Wilson's theorem.

$(4 \times 2 = 8)$

## Part D (Essay Questions)

*Answer any* **two** *questions.*
*Each question has weight* 4.

31 Determine whether the relation represented by the matrix

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$ is a partial order.

32 prove that if $n$ is an integer not divisible by 2 or 3, then $n^2 - 1$ is divisible by 24.

33 If P is a prime and $r$ is any number less than P–1, then prove that the sum of the products of the numbers 1, 2, 3, . . . . , P –1 taken $r$ together is divisible by P.

$(2 \times 4 = 8)$