E 8552 (Pages : 3) Reg. No.................................

Name.....................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2014

Fifth Semester

Core Course—ABSTRACT ALGEBRA

(Common for Model I and Model II B.Sc. Mathematics)

Time : Three Hours Maximum Weight : 25

Part A

Answer **all** *questions.*
Each bunch of 4 questions carries weight 1.

I. 1 On Q define a binary operation by $a*b = ab/2$. Determine whether * is commutative.

2 Give an example of an abelian group.

3 Define a cyclic group.

4 Every permutation is a one-one function. Write True *or* False.

II. 5 Find the number of generators of cyclic group of order 8.

6 G under addition is a cyclic group. State True *or* False.

7 Define the index of a subgroup H in a Group G.

8 What is the order of the coset $5 + < 4 >$ in the factor group $z_{12} / < 4 >$.

III. 9 Is the map $\phi : Z \to R$ under addition given by $n\phi = n,$ a homomorphism.

10 Define isomorphism of a ring R with a ring R′.

11 What are the units in the ring $z \times z$.

12 Find the characteristic of $z_3 \times z_4$.

IV. 13 Every field is an integral domain. Write True *or* False.

14 Define a quotient ring.

15 z_4 is an ideal of $4z$. Write True *or* False.

16 Is $z \times z$ an integral domain.

$(4 \times 1 = 4)$

Turn over

Part B

Answer any **five** *questions.*
Each question has weight 1.

17 Write the proper subgroup of S_3.

18 Compute $(1,2)(4,7,8)(2,1)(7,2,8,1,5)$.

19 Find the number of elements in the cyclic subgroup of z_{30} generated by 25.

20 Prove that an isomorphism maps the identity onto the identity.

21 If p is prime, show that z_p has no divisor of o.

22 Define a skew field. Give an example.

23 If R is a ring with unity and N is an ideal of R containing a unit. Show that N = R.

24 Find a subring of $z \times z$ that is not an ideal of $z \times z$.

(5 × 1 = 5)

Part C

Answer any **four** *questions.*
Each question has weight 2.

25 Let G be a group and $a \in G$. Show that $H = \{a^n / n \in Z\}$ is a subgroup of G and is the smallest subgroup of G that contains a.

26 Express $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 6 & 4 & 1 & 8 & 2 & 5 & 7 \end{pmatrix}$ as a product of disjoint cycles and then as a product of transpositions.

27 Prove that a factor group of a cyclic group is cyclic.

28 Prove that an infinite cyclic group G is isomorphic to the group Z of integers under addition.

29 Prove that every finite integral domain is a field.

30 Show that an intersection of ideals of a ring R is again an ideal of R.

(4 × 2 = 8)

Part D

Answer any **two** *questions.*
Each question has weight 4.

31 Let A be a non-empty set and let S_A be the collection of all permutations of A. Prove that S_A is a group under permutation multiplication.

32 State and prove Cayley's theorem.

33 Let ϕ be a homomorphism of a group G into a group G′ with Kernel K. Prove that $G\,\phi$ is a group and there is a canonical isomorphism of $G\,\phi$ with G/K.

$(2 \times 4 = 8)$