E 6281 (Pages : 3) Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2013

Fifth Semester

Core Course—ABSTRACT ALGEBRA

(Common for Model I and II B.Sc. Mathematics)

Time : Three Hours Maximum Weight : 25

Part A

Answer **all** *questions.*
Each bunch of four questions has weight of 1.

I. 1 Is the usual matrix addition a binary operation on M (R) set of all matrices with real entries ?

2 Whether the binary operation * defined on C by letting $a * b = |ab|$ gives a group structure on C.

3 Write a non-trivial proper sub group of Z_4.

4 Every permutation is a cycle. True or False.

II. 5 Find the quotient and remainder when –38 is divided by 7.

6 Every group of order less than or equal to 4 is cyclic. Write true *or* false.

7 Find the number of elements in the set $\{\sigma \in S_4 \mid \sigma(3) = 3\}$.

8 Every factor group of a cyclic group is cyclic. Write true *or* false.

III. 9 Define a skew field.

10 Is the set of all pure imaginary numbers with usual addition and multiplication a ring ?

11 Solve the equation $3x = 2$ in the field Z_7.

12 Find all units in Z_5.

IV. 13 Find the characteristics of the ring 2Z.

14 Every integral domain of characteristic zero is infinite – True *or* Flase.

15 Are $2Z/8Z$ and Z_4 isomorphic rings ?

16 Define Kernel of a ring homomorphism.

$(4 \times 1 = 4)$

Turn over

Part B

Answer any **five** *questions.*
Each question has weight 1.

17 Show that the binary structures (Q, t) and (Z, t) under usual addition are not isomorphic.

18 Let n be a positive integer and let $nz = \{nm\ (m \in Z\}$, show that $\langle nZ, t\rangle$ is a group.

19 Show that every cyclic group is abelian.

20 Show that every subgroup of an abelian group is normal.

21 Compute the product (16) (3) in Z_{32}.

22 Show that $M_2(Z_2)$ has zero divisors.

23 let R be a commutative ring with unity of characteristic 4. Compute and simplify $(a+b)^4$ for $a, b \in R$.

24 Let F be the ring of all functions mapping R into R and let C be the subring of F consisting of all the constant functions in F. Is C an ideal in F ? Why ?

$(5 \times 1 = 5)$

Part C

Answer any **four** *questions.*
Each question has weight 2.

25 Show that the identity element and inverse of each element are unique in a group.

26 Express $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 2 & 6 & 3 & 7 & 4 & 5 & 1 \end{pmatrix}$

as a product of disjoint cycles and as a product of transpositions.

27 Show that a subgroup of a cyclic group is cyclic.

28 Prove that M is a maximal normal subgroup of G if and only if G | M is simple.

29 If P is Prime, show that Zp has no divisors of 0.

30 $\phi : R \to R'$ be a ring homomorphism and let N be an ideal R. Show that ϕ (N) is an ideal of ϕ [R].

$(4 \times 2 = 8)$

Part D

Answer any **two** *questions.*
Each question has weight 4.

31 Prove that no permutation in Sn can be expressed both as a product of an even number of transpositions and as a product of a odd number of transpositions.

32 Let H be a subgroup of G. Prove that the left coset multiplication $(a\ H)(b\ H] = (ab)\ H$ is well defined if and only if H is a normal subgroup of G.

33 Prove that every integral domain is a field.

$(2 \times 4 = 8)$