E 4399 (Pages : 3) Reg. No.................................

Name.................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2012

Fifth Semester

Core Course—ABSTRACT ALGEBRA

(Common for Model-I and Model-II B.Sc. Mathematics)

Time : Three Hours Maximum Weight : 25

Part A (Objective Type Questions)

Answer **all** *questions.*
Each bunch of **four** *questions has weight* 1.

I. 1 Let $\mathbb{R}^*$ be the set of real numbers except zero. Define * on $\mathbb{R}^*$ by $a * b = |a|\, b$. Find a left identity for *.

2 Let S be the set of real numbers except –1. Define * on S by $a * b = a + b + ab$. Find the solution of $z * x = 8$ in S.

3 What is the order of the symmetric group S_n ?

4 Find the order of $\sigma\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ in S_3.

II. 5 Find the number of generators of a cyclic group of order 100.

6 Give an isomorphism from an abelian group G to itself.

7 Find all the left cosets of $2\mathbb{Z}$ in the group $\mathbb{Z}$ of integers under addition.

8 Give an example of a proper nontrivial normal subgroup of S_3.

III. 9 How many homomorphisms are there from the group $\mathbb{Z}$ of integers under addition on to itself?

10 Find the Kernel of the homomorphism $Q : \mathbb{Z} \to \mathbb{Z}/4\mathbb{Z}$ defined by $f(a) = a + 4\mathbb{Z},\ a \in \mathbb{Z}$.

11 Find all units in the ring $\mathbb{Q}$.

12 Find all divisors of zero in the ring $\mathbb{Z}_5$.

IV. State true or false :

13 Every integral domain of characteristic zero is infinite.

14 The characteristics of any field is zero.

15 Every ideal in a ring is a subring of the ring.

16 $2\mathbb{Z}$ is an ideal of the ring $\mathbb{Z}$.

$(4 \times 1 = 4)$

Turn over

Part B (Short Answer Type Questions)

Answer any **five** *questions.*
Each question has weight 1.

17 Show that every cyclic group is abelian.

18 Show by an example that every proper subgroup of a nonabelian group may be abelian.

19 Prove that the identity permutation is an even permutation.

20 Define the inner automorphism of a group G induced by the element g in G.

21 Define a division ring.

22 If R is a ring with unity, then show that this unity 1 is the only multiplicative identity.

23 Define factor ring of a ring.

24 An element a of a ring is idempotent if $a^2 = a$. Show that an integral domain has exactly two idempotent elements.

(5 × 1 = 5)

Part C (Short Essay Questions)

Answer any **four** *questions.*
Each question has weight 2.

25 Let G be a group and let a be a fixed element of G. Show that $H_a = \{x \in G : xa = ax\}$ is a subgroup of G.

26 Let A be a nonempty set. Prove that S_A, the collection of all permutations of A, is a group under permutation multiplication.

27 Let $\phi : G \to G'$, be a group isomorphism. Prove that :

(i) $e\varphi = e'$, where e and e' are identity elements of G and G′ respectively.

(ii) $(a^{-1})\varphi = (a\varphi)^{-1}$.

28 State and prove Lagrange's theorem.

29 Prove that every field is an integral domain. Also give an example of an integral domain which is not a field. Justify your claim.

30 Define an ideal of a ring R. If R is a ring with unity and N is an ideal of R containing a unit, prove that N = R.

(4 × 2 = 8)

Part D (Essay Questions)

Answer any **two** *questions.*
Each question has weight 4.

31 If $n \geq 2$, prove that the collection of all even permutations of $\{1, 2, \ldots\ldots, n\}$ form a subgroup of order $\frac{n!}{2}$ of the symmetric group S_n.

32 State and prove Cayley's theorem.

33 State and prove the fundamental homomorphism theorem for groups.

$(2 \times 4 = 8)$