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B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2015

Fifth Semester

Core Course—MATHEMATICAL ANALYSIS

(Common for Model I and Model II Mathematics and Computer Applications)

(2013 Admissions)

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all** *questions from this part.*
Each question carries 1 *mark.*

1. State Archimedean property of real numbers.

2. Write the infimum and supremum of the set $\left\{1+\frac{(-1)^n}{n}, n \in \mathrm{N}\right\}$.

3. Give an example of an open set which is not an interval.

4. Define limit point of a set.

5. Define a Cauchy sequence. Give an example.

6. Find $\lim_{n\to\infty} \frac{3+2\sqrt{n}}{\sqrt{n}}$.

7. Give an example of a sequence which oscillates finitely.

8. If $z_1 = -1$ and $z_2 = i$, find Arg $(z_1 z_2)$ where Arg z denotes the principal value of the argument of z.

9. If $z_1 = (-3, 1)$ and $z_2 = (1, 4)$ locate $z_1 + z_2$ vectorially.

10. Write $\frac{4+i}{2-3i}$ in the form $a + ib$.

$(10 \times 1 = 10)$

Part B

Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. Show that $|x+y| \le |x| + |y|$, for $x, y \in \mathrm{R}$.

12. Show that there is no rational number whose square is 2.

Turn over

13. Show that a nonempty finite set is not a *nbd* of any point.

14. Show that a set is closed if and only if its complement is open.

15. Show that for every set S, the closure $\overline{S}$ is closed.

16. Show that every convergent sequence in bounded.

17. Show that $\lim \dfrac{(3n+1)(n-2)}{n(n+3)} = 3$.

18. Give an example of two divergent sequences whose sum converges.

19. Show that the sequence $\left\{(-1)^n\right\}$ diverges.

20. Show that a sequence of real numbers can have at most one limit.

21. Show that $\operatorname{lm}(iz) = nez$.

22. Write $-1-i$ in exponential form.

(8 × 2 = 16)

Part C

Answer any **six** *questions.*
Each question carries 4 marks.

23. Write the properties of R which makes it info a complete-ordered field.

24. Show that set of rational numbers is not order complete.

25. Show that the union of arbitrary family of open sets is open.

26. Show that intersection of an arbitrary family of closed sets is closed.

27. Show that deviced set of a bounded set is bounded.

28. Show that the supremum of a bounded non empty set $S \subset R$, when not a member of S is a limit point of S.

29. Use Cauchy's general principle of convergence to show that the sequence $\left\{\dfrac{n}{n+1}\right\}$ is convergent.

30. Show that for any number x, $\lim \dfrac{x^n}{n!} = 0$.

31. Show that $\left|n_e\left(2+\overline{z}+z^3\right)\right| \leq 4$ when $|z| \leq 1$.

(6 × 4 = 24)

Part D

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) Show that the set of rational numbers in $[0, 1]$ is countable.

    (b) State and prove Bolzano weierstrass theorem.

33. (a) Show that interior of a set S is the largest open subset of S.

    (b) Show that **the derived** set S' of a bounded infinite set S has the smallest and the greatest members.

34. (a) Show that $\lim_{n\to\infty} \sqrt[n]{n} = 1$.

    (b) State and prove Cauchy's first theorem on limits.

35. (a) State and prove nested interval property of real numbers.

    (b) Show that the sequence $\{s_n\}$ where $s_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \ldots\ldots + \frac{1}{(n-1)!}$ is convergent.

(2 × 15 = 30)