E 8550 (Pages : 3) Reg. No.................................

Name...................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2014

Fifth Semester

Core Course—MATHEMATICAL ANALYSIS

(Common For Model I and Model II Mathematics and Computer Applications)

Time : Three Hours Maximum Weight : 25

Part A

Answer **all** *questions.*
Each bunch of **four** *questions has weight* 1.

I. 1 Find the supremum of the set $\left\{\frac{(-1)^n}{n}, n \in \mathrm{N}\right\}$.

2 State Archimedean property of real numbers.

3 Is the set $\left\{\frac{1}{n}, n \in \mathrm{N}\right\}$ open.

4 Find the derived set of $\left\{\frac{1}{m}+\frac{1}{n}, m, n \in \mathrm{N}\right\}$

II. 5 Give an example of a perfect set.

6 Show that the set $\left\{1, -1, 1\frac{1}{2}, -1\frac{1}{2} \ldots.\right\}$ is closed.

7 Give an example of a bounded set having infinite number of limit points.

8 Define closure of a set.

III. 9 Find the limit points of the sequence $\{S_n\}$, where $S_n = (-1)^n, n \in \mathrm{N}$.

10 Whether the sequence $\left\{m+\frac{1}{n}, m, n \in \mathrm{N}\right\}$ converges.

11 Find $\lim_{n \to \infty} \frac{x^n}{n!}, x \in \mathrm{R}$.

12 Define a Cauchy sequence.

Turn over

IV. 13 What is a monotonic sequence ?

14 When does $\{r^n\}$ converges ?

15 What is the value of $e^{-i\pi/2}$.

16 Is it true that $R(iz) = -Imz$.

(4 × 1 = 4)

Part B

Answer any **five** *questions.*
Each question has weight 1.

17 Prove that the greatest number of a set if it exists is the supremum of the set.

18 If $a \in R$ and $a \neq 0$, then show that $a^2 > 0$.

19 Show that finite union of closed sets is closed.

20 Show that closure of a set is a closed set.

21 Show that $\lim_{n \to \infty} n\sqrt{n} = 1$.

22 Show that $\{S_n\}$, where $S_n = 1 + \frac{1}{2} + .. + \frac{1}{n}$ cannot converge.

23 Prove that $\lim_{n \to \infty} \left\{ \frac{1 + \frac{1}{2} + \ldots.. + \frac{1}{n}}{n} \right\} = 0$.

24 Sketch the set $|2z + 3| > 4$.

(5 × 1 = 5)

Part C

Answer any **four** *questions.*
Each question has weight 2.

25 Show that set of rational numbers is not order complete.

26 Prove that subset of a countable set is countable.

27 Show that every open set is the union of open intervals.

28 Prove that a necessary and sufficient condition for the convergence of a monotone sequence is that it is bounded.

29 Show that $S_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \ldots + \frac{1}{(n-1)!}$ is convergent.

30 Find all the roots of $(-1)^{1/3}$ in rectangular co-ordinate system. Exhibit them as the vertices of a regular polygon.

$(4 \times 2 = 8)$

Part D

Answer any **two** *questions.*
Each question has weight 4.

31 State and prove Bolzano-Weierstrass theorem for sets.

32 State and prove Cauchy's first theorem on limits.

33 State and prove Cauchy's general principle of convergence.

$(2 \times 4 = 8)$