E 6279 (Pages : 3) Reg. No..............................

Name..................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2013

Fifth Semester

Core Course—MATHEMATICAL ANALYSIS

(Common for Model I and II Mathematics and Computer Applications)

Time : Three Hours Maximum Weight : 25

Part A (Objective Type Questions)

Answer **all** *the questions.*
Each bunch of **four** *questions has weight* 1.

I. 1. Find the infimum of $\left\{\frac{(-1)^n}{n},\ n \in \mathrm{N}\right\}$.

2. What is order completeness property of R ?

3. The set Q of rationals is an open set. True *or* False.

4. Find the derived set of $\left\{1+\frac{1}{n},\ n \in \mathrm{N}\right\}$.

II. 5. Give an example of a nowhere dense set.

6. Give an example of a dense subset of R, reals.

7. Give an example of a bounded set having no limit point.

8. State Bolzano Weierstrass theorem for sets.

III. 9. Find the limit points of $\{S_n\}$ where $S_n = 1+(-1)^n,\ n \in \mathrm{N}$.

10. Whether the sequence $\{n(-1)^n\}$ converges.

11. Define a Cauchy sequence.

12. Give an example of a sequence which is cauchy.

IV. 13. State Cauchy's general principle of convergence.

14. Give an example of a monotonic increasing sequence which converges.

15. Show that 1m (iz) = Rez.

16. Express $-1-i$ in exponential form.

$(4 \times 1 = 4)$

Turn over

Part B (Short Answer Questions)

Answer any **five** *questions.*

Each question carries weight 1.

17. Show that a set cannot have more than one supremum.
18. State Dedekind's property of real numbers.
19. If M and N are neighborhoods of a point x, show that $M \cap N$ is also a neighborhood of x.
20. Show that set of rational numbers in [0, 1] is countable.
21. Show that every convergent sequence is bounded.
22. Show that $\lim \frac{1+3+5+...+(2n-1)}{n^2} = 1$.
23. Show that for any x, $\lim \frac{x^n}{n!} = 0$.
24. Prove that $|z_1 + z_2|^2 \leq |z_1|^2 + |z_2|^2$.

(5 × 1 = 5)

Part C (Short Essay Questions)

Answer any **four** *questions.*

Each question has weight 2.

25. Show that the real number field is Archimedian.
26. Prove that countable collection of countable sets is countable.
27. Prove that interior of a set is the largest open subset in it.
28. Find $\lim_{n \to \infty} \left[\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + ... + \frac{1}{\sqrt{n^2+n}} \right]$.
29. Show that a sequence cannot converge to more than one limit.
30. Find all values of $(-8i)^{1/3}$.

(4 × 2 = 8)

Part D (Essay Questions)

Answer any **two** *questions.*
Each question has weight 4.

31. Prove that set of all real numbers in $[0,1]$ is uncountable.

32. Prove that $\{r^n\}$ converges if and only if $-1 < r \leq 1$.

33. State and prove Bolzano-Weierstrass theorem for sequences.
    Is the converse true. Justify.

$(2 \times 4 = 8)$