E 3208 (Pages : 3) Reg. No.....................................

Name.......................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2016

Fifth Semester

Core Course—MATHEMATICAL ANALYSIS

(Common for Model I and Model II Mathematics and Computer Applications)

[2013 Admission onwards]

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all** *questions from this part.*
Each question carries 1 *mark.*

1. Give an example of a set which is bounded above but not bounded below.
2. Define the infimum of a non-empty subset of R.
3. Is the set $\left\{\frac{n}{1}, n \in \mathrm{N}\right\}$ open ?
4. What is a perfect set ?
5. Is every bounded sequence converges.
6. What is the limit of the sequence $\left\{1+\frac{1}{n}\right\}$?
7. Give an example of a sequence which oscillates finitely.
8. State de Moivre's Formula.
9. Define argument and Principal argument of a complex number.
10. Define connected set in the complex plane.

(10 × 1 = 10)

Part B

Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. Give examples to show that the supremum and infimum of a set may not belong to the set.
12. State Dedekind's property.

Turn over

13. Show that there is no rational number whose square is 2 ?

14. Find the derived set of the following :—

(a) $\{x : 0 < x < 1, \ x \in \mathbb{Q}\}$.

(b) $\left\{\frac{1}{m}+\frac{1}{n}, \ m, n \ \in \mathbb{N}\right\}$.

15. Give examples for each of the following :—

(a) A set which is neither open nor closed.

(b) A set which is not an interval and is closed.

16. Show that the set of all even natural numbers is countable.

17. Show that if $\lim a_n = a$ and $a_n \geq 0$ for all n, then $a \geq 0$.

18. Show that the sequence $[s_n]$, where $s_n = \frac{1}{n+1}+\frac{1}{n+2}+\ldots+\frac{1}{n+n}$ convergent.

19. Find the limit of the sequence $\left\{\left(2+\frac{1}{n}\right)^2\right\}$.

20. Sketch the set and determine whether it is a domain $|z-2+i| \leq 1$.

21. Sketch the set of points determined by the condition $|2\bar{z}+i| = 4$.

22. Locate the complex numbers $z_1 + z_2$ and $z_1 - z_2$ vectorially when $z_1 = z_i$ and $z_2 = \frac{2}{3} - i$.

(8 × 2 = 16)

Part C

Answer any **six** *questions.*
Each question carries 4 *marks.*

23. State and prove Archimedean Property of Real Numbers.

24. Prove that the greatest member of a set if it exists is the supremum of the set.

25. Show that every open interval is an open set.

26. Show that the set of all Rational Numbers in [0, 1] is countable.

27. Use Cauchy's general principle of Convergence to show that $\left\{\frac{n}{n+1}\right\}$ is Convergent.

28. Show that the sequence $\{bn\}$ where $\{bn\} = \left[\frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \ldots + \frac{1}{(2n)^2}\right]$ converges to zero.

29. Show that $\lim_{n \to \infty} \sqrt[n]{n} = 1$.

30. Find all values of $(-8i)^{1/3}$.

31. Find the principal argument Argz of $z = \frac{-2}{1 + \sqrt{3}\, i}$.

(6 × 4 = 24)

Part D

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) Prove that the intersection of an arbitrary family of closed sets is closed.

(b) Prove that countable union of countable sets is countable.

33. (a) If S and T are sets of real numbers show that (i) $S \subset T \Rightarrow S' \subset T'$ and (ii) $(S \cup T)' = S' \cup T'$, where S' denotes the derived set of S.

(b) Prove that derived set of a set is closed.

34. State and prove Cauchy's General Principle of Convergence.

35. (a) Show that $\{s_n\}$ were $s_n = \frac{1}{1!} + \frac{1}{2!} + \ldots + \frac{1}{n!}$ is convergent.

(b) Show that $\lim_{n \to \infty} \left\{1 + 2^{1/2} + 3^{1/3} + \ldots n^{1/n}\right\} = 1$.

(2 × 15 = 30)