E 1346 (Pages : 3) Reg. No..................................

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B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2015

Third Semester

Complementary Course—Mathematics

VECTOR CALCULUS, DIFFERENTIAL EQUATIONS AND ANALYTIC GEOMETRY

(Common for B.Sc. Physics, Chemistry, Petrochemicals, Geology, Computer Maintenance and Electronics and Food Science and Quality Control)

[2013 Admission onwards]

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all** *questions from this part.*
Each question carries 1 *mark.*

1. Find the length of $u(t) = (\sin t)\, i + (\cos t)\, j + \sqrt{3}\, k$.
2. $r(t)$ is the position of a particle in space at time t. Find the velocity of the particle at $t = 1$, where $r(t) = (t+1)\, i + (t^2 - 1)\, j + 2\, t\bar{k}$.
3. Find the unit tangent vector to the curve $r(t) = (2+t)\, i - (t+1)\, j + tk, 0 \le t \le 3$.
4. Find the gradient field of $g(x, y, z) = xy + yz + xz$.
5. State Green's theorem.
6. State Gauss's divergence theorem.
7. Find an integrating factor of $(y - 2x^3)\, dx - x(1 - xy)\, dy = 0$.
8. Write the general form of a first order linear differential equation.
9. Find the directrices of the ellipse $7x^2 + 16\, y^2 = 112$.
10. Write the parametric equation of a cycloid generated by a circle of radius a.

$(10 \times 1 = 10)$

Part B

Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. Evaluate $\int_0^{\pi} ((\cos t)\, i + j - 2tk)\, dt$.
12. Find T and N for the circular motion $r(t) = (\cos 2t)\, i + (\sin 2t)\, j$.

Turn over

13. Write a in the form $a = a_T T + a_N N$ without finding T and N, where $r(t) = (2t + 3)i + (t^2 - 1)j$.

14. Evaluate $\int_C (x - y + z - 2)\, ds$ where C is the straight line segment $x = t$, $y = (1 - t)$, $z = 1$ from $(0, 1, 1)$ to $(1, 0, 1)$.

15. Find the gradient of $f(x, y, z) = \ln\sqrt{(x^2 + y^2 + z^2)}$.

16. Test whether $F = yzi + xzj + xyk$, conservative.

17. Solve $e^y\, dx + (xe^y + 2y)\, dy = 0$.

18. Solve $\frac{dy}{dx} - y \tan x = \sin x$.

19. Solve $y + px = p^2 x^4$.

20. Sketch the parabola $y^2 = -2x$.

21. Find the eccentricity of $x^2 - y^2 = 1$. Also find the foci and directrices.

22. Find the tangent to the right-hand hyperbola branch $x = \sec t, y = \tan t, \frac{-\pi}{2} 2t < \frac{\pi}{2}$ at the point $(\sqrt{2}, 1)$, where $t = \pi/4$.

$(8 \times 2 = 16)$

Part C

Answer any **six** *questions.*
Each question carries 4 marks.

23. Find the point on the curve $r(t) = (5 \sin t)i + (5 \cos t)j + 12tk$ at a distance 26π units along the curve from the origin in the direction of increasing arc length.

24. Find T, N and K for the plane curve $r(t) = (\ln \sec t)i + tj, \frac{-\pi}{2} < t < \frac{\pi}{2}$.

25. Find the work done by $F = 6zi + y^2 j + 12xk$ over the curve $r(t) = (\sin t)i + (\cos t)j + \left(\frac{t}{6}\right)k$, $0 \le t \le 2\pi$.

26. Show that $F = (e^x \cos y + yz)i + (xz - e^x \sin y)j + (xy + z)k$ is conservative and find a potential function for it.

27. Solve $p^2 + xp - y = 0$.

28. Solve $(3x + 2y^2)\, y\, dx + 2x\,(2x + 3y^2)\, dy = 0$.

29. Find the general and singular solutions of $py = xp^2 + a$.

30. Sketch the conic $r = \dfrac{4}{2 - 2\cos\theta}$.

31. Find a Cartesian equation for the hyperbola centered at the origin that has a focus at (3, 0) and the line $x = 1$ as the corresponding directrix.

$(6 \times 4 = 24)$

Part D

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. Find the curvature of the helix $r(t) = (a\cos t)\, i + (a \sin t)\, j + btk,\ a, b \geq 0,\ a^2 + b^2 \neq 0$. Also find N for the helix.

33. Find the area of the cap cut from the hemisphere $x^2 + y^2 + z^2 = 2,\ z \geq 0$, by the cylinder $x^2 + y^2 = 1$.

34. Use the surface integral in Stoke's theorem to calculate the circulation of the field $\mathbf{F} = x^2 i + 2\, xj + z^2 k$, around the curve C, which is the ellipse $4x^2 + y^2 = 4$ in the xy-plane, counter clockwise viewed from above.

35. Using divergence theorem find the outward flux of $\mathbf{F} = x^2 i + xzj + 3zk$ across the boundary of the region, D, the solid sphere $x^2 + y^2 + z^2 \leq 4$.

$(2 \times 15 = 30)$