E 6151 (Pages : 3) Reg. No.................................

Name.................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2013

Third Semester

Complementary Course—Mathematics

VECTOR, CALCULUS, DIFFERENTIAL EQUATIONS AND ANALYTIC GEOMETRY

(Common for Physics, Chemistry, Petrochemicals, Geology, Computer Maintenance and Electronics and Food Science and Quality Control)

(2011 Admission onwards)

Time : Three Hours Maximum Weight : 25

Part A

Answer **all** *questions.*

Each bunch of **four** *questions carries a weight of* 1.

I. 1. Give an example of a discontinuous vector function.

2. Find the unit tangent vector of the helix $r(t) = \cos t\, i + \sin t\, j + t\, k$.

3. Define torsion of a smooth curve.

4. Find the direction in which $f(x, y) = \frac{x^2}{2} + \frac{y^2}{2}$ increases most rapidly.

II. 5. Find the gradient field of $g(x, y, z) = xy + yz + xz$.

6. Show that $F = (2x - 3)i - 2j + (\cos z)k$ is not conservative.

7. Find the curl of $F(x, y) = (y^2 - x^2)i + (xy - 2y)j$.

8. State Green's theorem.

III. 9. Is the equation $(\cos x - x\cos y)\frac{dy}{dx} = \sin y + y\sin x$ exact.

10. Write the general form of a first order linear equation.

11. What are integrating factors.

12. Write an integrating factor of $ydx - xdy + (x^2 + y^2)dx = 0$.

Turn over

IV. 13. Write the equation of the ellipse in standard form whose foci are $(0, \pm 4)$ and vertices $(0, \pm 5)$.

14. Write the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

15. Which conic is represented by the equation $x^2 + 2xy + y^2 + 2x - y + 2 = 0$.

16. Write the polar equation of the hyperbola with eccentricity 3/2 and directrix $x = 2$.

$(4 \times 1 = 4)$

Part B

Answer any **five** *questions.*
Each question has weight 1.

17. Find the principal unit normal N for the helix $r(t) = a\cos t\, i + a\sin t\, j + btk,\ a, b \geq 0,$ $a^2 + b^2 \neq 0$.

18. Find the derivative of $f(x, y) = xe^y + \cos(xy)$ at the point $(2, 0)$ in the direction of $A = 3i - 4j$.

19. Evaluate $\int_C (x - y + z - 2)\, ds$ where C is the straight line segment $x = t,\ y = 1 - t,\ z = 1$, from $(0, 1, 0)$ to $(1, 0, 0)$.

20. Calculate the outward flux of the field $F(x, y) = xi + y^2 j$ across the square bounded by the lines $x = \pm 1,\ y = \pm 1$.

21. Solve the equation $y = 2px + y^2 p^3$.

22. Solve the equation $\frac{dy}{dx} + y\cot x = e^x$.

23. Find the center, foci, vertices and asymptotes of the hyperbola $\frac{(x-2)^2}{16} - \frac{y^2}{9} = 1$.

24. Describe the motion of a particle whose position $p(x, y)$ at time t is given by $x = a\cos t,\ y = b\sin t\ 0 \leq t \leq 2\pi$.

$(5 \times 1 = 5)$

Part C (Short Essays)

Answer any **four** *questions.*
Each question has weight 2.

25. Find the tangent plane and normal line of the surface $f(x, y, z) = x^2 + y^2 + z - 9 = 0$ at the point $(1, 2, 4)$.

26. Evaluate $\int_{-1}^{0} \int_{-1}^{1} (x + y + 1)\, dx\, dy$.

27. Find the center of the mass of a thin shell of constant density δ cut from the cone $z = \sqrt{x^2 + y^2}$ by the planes $z = 1$ and $z = 2$.

28. Find the integrating factor and solve the equation
$(x \cos y - y \sin y) dy + (x \sin y + y \cos y) dx = 0$.

29. Sketch the parabola $(y + 2)^2 = 8(x - 1)$. Plot the vertex, focus and directrix.

30. Rotate the co-ordinate axes through an angle α to remove the xy term from the equation $2x^2 + \sqrt{3}\, xy + y^2 - 10 = 0$. Find α and identify the new curve.

$(4 \times 2 = 8)$

Part D

Answer any **two** *questions.*
Each question has weight 4.

31. Use Stoke's theorem to calculate the circulation of the field $\mathrm{F} = 2yi + 3xj - z^2 k$ around the curve $\mathrm{C} = x^2 + y^2 = 9$ in the xy plane, counterclockwise.

32. (a) Solve the equation $(px - y)\ (py + x) = 2p$.

(b) Solve $4\dfrac{dy}{dx} - y \tan x + y^5 \sin 2x = 0$.

33. (a) Find a polar equation of the conic with $e = \dfrac{1}{2}$, one focus at the origin and directrix $x = 1$ corresponding to that focus.

(b) Describe the motion of a particle whose position $p(x, y)$ at time t is given by $x = \sec t,\ y = \tan t, -\pi/2 < t < \pi/2$.

$(2 \times 4 = 8)$