E 3979 (Pages : 4) Reg. No..............................

Name..................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2016

Third Semester

Complementary Course—Mathematics

VECTOR CALCULUS, DIFFERENTIAL EQUATIONS AND ANALYTIC GEOMETRY

(Common for B.Sc. Physics, Chemistry, Petrochemicals Geology Computer Maintenance and Electronics and Food Science and Quality Control)

[2013 Admission onwards]

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all** *questions from this part.*
Each question carries 1 *mark.*

1. Show that curvature of a straight line is zero.
2. $r(t) = 3\cos t\, i + 3\sin t\, j + t^2\, k$ gives the position of a moving body at time t. Find the acceleration of the body at time t.
3. Define gradient field of a differentiable function $f(x, y, z)$.
4. Define a conservation field.
5. State normal form of Green's theorem.
6. How will you define flux of a three dimensional vector field across on oriented surface S ?
7. Find an integrating factor of the differential equation $\left(y - 2x^3\right) dx - x\left(1 - xy\right) dy = 0$.
8. Define an exact differential equation.
9. Find the asymptotes of the hyperbola $\frac{y^2}{4} - \frac{x^2}{5} = 1$.
10. Find the eccentricity of the ellipse $9x^2 + 10y^2 = 90$.

$(10 \times 1 = 10)$

Turn over

Part B

Answer any **eight** *questions.*
Each question carries 2 marks.

11. Find the unit tangent vector to the curve $r(t) = t\cos t\, i + t\sin t\, j + \left(2\sqrt{2}/3\right)t^{3/2}\, k, 0 \le t \le \pi.$

12. Find the binomial vector B for the curve $r(t) = (\cos t + t\sin t)\, i + (\sin t - t\cos t)\, j + 3\bar{k}.$

13. Find ∇f at (1, 1, 1) where $f(x, y, z) = 2z^3 - 3\left(x^2 + y^2\right)z + \tan^{-1} xz.$

14. Evaluate $\int_C \sqrt{x^2 + y^2}\, ds$ along the curve $r(t) = (4\cos t)\, i + (4\sin t)\, j + 3tk, -2\pi \le t \le 2\pi.$

15. Find the work done by $\mathbf{F} = xyi + yj - yzk$ over the curve $r(t) = ti + t^2 j + tk, 0 \le t \le 1.$

16. Find the curl of $\mathbf{F} = \left(x^2 - y\right)i + 4zj + x^2 k.$

17. Solve the equation $p^2 + 2py\cot x = y^2$, where $p = \frac{dy}{dx}.$

18. Solve $\frac{dy}{dx} + y\tan x = \sec x.$

19. Solve $y = p\sin p + \cos p.$

20. Find the focus and directrix of the parabola $y = -8x^2.$

21. Sketch the ellipse $2x^2 + y^2 = 2.$

22. Find the polar equation of the circle $(x - 6)^2 + y^2 = 36.$

$(8 \times 2 = 16)$

Part C

Answer any **six** *questions.*
Each question carries 4 *marks.*

23. Find the curvature for the helix $r(t) = (a \cos t)\, i + (a \sin t)\, j + bt\, \bar{k},\, a, b \geq 0,\, a^2 + b^2 \neq 0$.

24. Estimate how much the value of $f(x, y, z) = xe^y + yz$ will change if the point $p(x, y, z)$ moves 0.1 unit from $p_0\,(2, 0, 0)$ straight toward $p_1\,(4, 1, -2)$.

25. Find the flux of $F = (x - y)\, i + xj$ across the circle $x^2 + y^2 = 1$ in the xy plane.

26. Find the flux of $F = yzj + z^2k$ outward through the surface s cut from the cylinder $y^2 + z^2 = 1, z \geq 0$ by the plar es $x = 0$ and $x = 1$.

27. Find a parametrization of the sphere $x^2 + y^2 + z^2 = a^2$.

28. Solve $y^2\,(y - xp) = x^4\, p^2$.

29. Solve $(\cos x - x \cos y)\dfrac{dy}{dx} = \sin y + y \sin x$.

30. Derive the standard equation of the parabola.

31. Sketch the hyperbola $8y^2 - 2x^2 = 16$ include asymptotes and foci in your sketch.

$(6 \times 4 = 24)$

Part D

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) Find the directional derivative of $f(x, y, z) = xy + yz + zx$ at $p_0\,(1, -1, 2)$ in the direction of $A = 3i + 6j - 2k$.

    (b) Write a in the form $a = a_T\, T + a_N\, N$ at the given value of t without finding T and N : $r(t) = t \cos t\, i + t \sin t\, j + t^2\, k, t = 0$.

Turn over

33. Find the area of the cap cut from the hemisphere $x^2 + y^2 + z^2 = 2, z \geq 0$, by the cylinder $x^2 + y^2 = 1$.

34. Use the surface integral in Stoke's theorem to calculate the circulation of the field $F = 2yi + 3xj - z^2k$ around the curve C : the circle $x^2 + y^2 = 9$ in the xy plane, counter clockwise when viewed from above.

35. (a) Sketch the conic $r = \dfrac{6}{2 + \cos\theta}$.

(b) Find a polar equation for an ellipse with semimajor axis 39.44 AU and eccentricity 0.25.

(2 × 15 = 30)