E 1596 (Pages : 3) Reg. No.....

Name.....

C.B.C.S.S. – B.Sc. DEGREE EXAMINATION, APRIL 2011

Fourth Semester

MP4 CO1—DIFFERENTIAL EQUATIONS, ABSTRACT ALGEBRA, NUMERICAL ANALYSIS

(Complementary Course to Physics/Chemistry/Petrochemical/Geology/Food Science and Quality Control/Computer Maintenance and Electronics)

Time : Three Hours Maximum Weight : 25

Part A (Objective Type Questions)

Answer **all** *questions.*
Each bunch of 4 *questions has weight* 1.

I. 1 Find a solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$.

2 What is the total differential of a function $u(x, y)$ of two variables x and y ?

3 Find an integrating factor of the differential equation $\frac{dy}{dx} + y = x$.

4 Give an example of a homogeneous differential equation.

II. 5 Give an example of a three dimensional surface by giving its equation.

6 What is the order of the partial differential equation $\left(\frac{\partial\theta}{\partial x}\right)^3 + \frac{\partial^2\theta}{\partial t^2} = 0$.

7 Form a partial differential equation by differentiating once with respect to x from the surface $x^2 + y^2 + (z - c)^2 = a^2$, where a and c are arbitrary.

8 Write the general form of a linear partial differential equation.

III. 9 If $u = 3v^7 - 6v$, find the percentage error in u at $v =$ if the error in v is 0.05.

10 Find *two* values a and b such that a real root of $f(x) = x^3 - x - 1 = 0$ lies between a and b.

11 Name the method in which we replace the curve between $(x_0, f(x_0))$ and $(x_1, f(x_1))$ by means of the chord joining these points to find a root of $f(x) = 0$ in between x_0 and x_1.

12 In Newton Raphson method, what is the formula to find the $(n + 1)^{th}$ approximation x_{n+1} to a root of $f(x) = 0$ if the n^{th} approximation x_n is known ?

Turn over

IV. 13 Give an example of a group of order 9.

14 Write the subgroup of $\mathbb{Z}_5$ generated by 2.

15 How many elements are there in the group of symmetries of a square ?

16 Give an example of a group homomorphism from $\mathbb{Z}$ to $3\mathbb{Z}$.

$(4 \times 1 = 4)$

Part B (Short Answer Questions)

Answer any **five** *questions.*
Each question has weight 1.

17 Find the general solution of the differential equation $x\frac{dy}{dx} + 2y = 3$.

18 Solve the differential equation $\left(\frac{dy}{dx}\right)^3 = \left(\frac{dy}{dx}\right)e^{2x}$.

19 Eliminate the arbitrary function f from the equation $z = xy + f\left(x^2 + y^2\right)$.

20 Find $\frac{\partial(F, G)}{\partial(y, z)}$ if $F = ax^2 + by^2 + cz^2 - 1$ and $G = x + y + z - 1$.

21 Find an approximate root of $x^3 - 18 = 0$ using bisection method.

22 Find an approximate root of $\cos x = 3x - 1$ using the interative method.

23 Find the cyclic subgroups $\langle \rho_1 \rangle$ and $\langle \mu_1 \rangle$ of S_3.

24 Give an example of a vector space over the field $\mathbb{R}$ by properly defining addition and scalar multiplication.

$(5 \times 1 = 5)$

Part C (Short Essay Questions)

Answer any **four** *of the following questions.*
Each questions has weight 2.

25 Check whether the following differential equation is exact or not. If exact, solve it :—

$$\left(2xe^y - x\cos(xy) + 2y\right)dy - \left(y\cos(xy) - e^{2y}\right)dx = 0.$$

26 Solve the differential equation $\left(x^2 + y^2\right)dx = \left(xy - x^2\right)dy$.

27 Find the general solution of the differential equation :

$$x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = (x + y)z.$$

28 Find the integral curves of the equations :

$$\frac{dx}{y(x + y) + az} = \frac{dy}{x(x + y) - az} = \frac{dz}{z(x + y)}.$$

29 Use Newton-Raphson method to find a real root of the equation $x^3 - 3x - 5 = 0$.

30 Show that the intersection of the subgroups H_i of a group G for $i \in I$ is again a subgroup of G.

$(4 \times 2 = 8)$

Part D (Essay Questions)

Answer any **two** *questions.*
Each question has weight 4.

31 Solve $\frac{dy}{dx} + \frac{1}{x}y = 3y^3$.

32 Find a real root of the equation $f(x) = x^3 - 2x - 5 = 0$ using the method of false position.

33 Show that the set of all complex numbers with usual addition and multiplication is a field.

$(2 \times 4 = 8)$