E 6838 (Pages : 4) Reg. No..............................

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B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, NOVEMBER 2013

First Semester

Complementary Course—DIFFERENTIAL CALCULUS AND TRIGONOMETRY

(Complementary Course for Physics/Chemistry/Petrochemicals/Geology Food Science and Quality Control/Computer Maintenance and Electronics)

[2013 Admissions]

Time : Three Hours Maximum : 80 Marks

Part A

Short Answer Questions. (Answer **all** *questions).*
Each question carries 1 mark.

1. Find $\lim_{x \to 1} \frac{x^2-1}{x-1}$.
2. Define the instantaneous rate of change of a function with respect to x.
3. State the extreme value theorem.
4. Define critical point of a function.
5. State Rolle's theorem.
6. What is the physical interpretation of the mean value theorem ?
7. Define level curve of a function $f(x, y)$.
8. Find $\frac{\partial f}{\partial y}$ if $f(x, y) = \sqrt{x^2 + y^2}$.
9. Define the hyperbolic sine of x.
10. What is the period of $\cosh(x + iy)$?

$(10 \times 1 = 10)$

Part B

Brief Answer Questions. (Answer any **eight** *questions).*
Each question carries 2 marks.

11. If $3 - x^3 \leq f(x) \leq 3 \sec x$ for all x, find $\lim_{x \to 0} g(x)$.
12. Find the slope of the curve $y = x + \frac{2}{x}$ at $x = 1$.

Turn over

13. If $x = 2t + 3$ and $y = t^2 + 1$, find the value of $\frac{dy}{dx}$ at $t = 6$.

14. Find the absolute minimum of $g(t) = 8t - t^4$ on $[-2, 1]$.

15. Show that the function $f(x) = x^4 + 3x + 1$ has exactly one zero in $[-2, 1]$.

16. Given that the velocity $v(t) = 9.8t + 5$, $s(0) = 10$. Find the position $s(t)$ of the body at time t.

17. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial t}{\partial y}$ if $f(x, y) = x^2 y + \cos y + y \sin x$.

18. Use chain rule to find $\frac{dw}{dt}$ if $w = x^2 + y^2$, $x = \cos t$, $y = \sin t$. Also find $\frac{dw}{dt}$ at $t = \frac{\pi}{2}$.

19. Using implicit differentiation formula find $\frac{dy}{dx}$ if $y^2 - x^2 - \sin xy = 0$.

20. Define $\sin x$ and $\cos x$ interms of exponential functions and show that $\sin(x + y) = \sin x \cos y + \cos x \sin y$.

21. Prove that $\cosh 3x = 4\cosh^3 x - 3\cosh x$.

22. Show that $\cosh^{-1} x = \log(x + \sqrt{x^2 - 1})$.

$(8 \times 2 = 16)$

Part C

Short Essay Questions. (Answer any **six** *questions).*
Each question carries 4 marks.

23. Using the Sandwich Theorem find the horizontal asymptote of $y = 2 + \frac{\sin x}{x}$.

24. Find $\frac{d^2y}{dx^2}$ as a function of t if $x = t + \frac{1}{t}$, $y = t - \frac{1}{t}$.

25. Use implicit differentiation to find $\frac{dy}{dx}$ and then find $\frac{d^2y}{dx^2}$ if $x^{2/3} + y^{2/3} = 1$.

26. If $f'(x) = 0$ at each point of an open interval (a, b), then prove that $f(x)$ is constant for all x in (a, b).

27. Find the critical points of $f(x) = x^3 - 12x - 5$ and identity the intervals on which f is increasing and decreasing.

28. Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if :

$$w = x^2 + y^2,\ x = r - s,\ y = r + s.$$

29. If resistors of R_1, R_2 and R_3 ohms are connected in parallel to make an R–ohm resistor, the value of R can be found from $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$. Find the value of $\frac{\partial R}{\partial R_2}$ when $R_1 = 30$, $R_2 = 45$ and $R_3 = 90$ ohms.

30. Separate into real and imaginary parts of $\tan(\alpha + i\beta)$.

31. Express $\frac{\sin 6\theta}{\sin\theta}$ in terms of $\cos\theta$. $(6 \times 4 = 24)$

Part D

Essay Questions. (Answer any **two** *questions).*
Each question carries 15 *marks.*

32. (a) Let $f(x) = \sqrt{x-1}$ and $\varepsilon = 1$. Find a $\delta > 0$ such that for all x with $0 < |x-5| < \delta$ the inequality $|f(x) - 2| < \varepsilon$ holds.

(b) Does the curve $y = x^4 - 2x^2 + 2$ have any horizontal tangents ? If so where ?

(c) Find the equation of the tangent at $t = \frac{\pi}{4}$ to the curve whose parametric equations are $x = \cos t$, $y = \sin t$, $0 \le t \le 2\pi$.

33. (a) State and prove the mean value theorem.

(b) For what values of a, m and b does the function :

$$f(x) = \begin{cases} 3, & x = 0 \\ -x^2 + 3x + a, & 0 < x < 1 \\ mx + b & 1 \le x \le 2 \end{cases}$$

Satisfy the hypothesis of the mean value theorem on the interval [0, 2] ?

Turn over

(c) Find the critical points of the function f whose derivative $f'(x) = x(x-1)$. Identity the intervals on which f is increasing and decreasing. Also find the local extreme values.

34. (a) Verify that $f_{xy} = f_{yx}$ if

$f(x, y) = \log(2x + 3y)$.

(b) Find $\frac{dw}{dt}$ at $t = 0$ if

$w = x^2 + y^2,\; x = \cos t + \sin t,\; y = \cos t - \sin t$.

(c) Find $\frac{\partial w}{\partial r}$ when $r = 1, s = -1$ if

$w = (x + y + z)^2, x = r - s, y = \cos(r + s)\; z = \sin(r + s)$.

35. (a) Expand $\sin^7\theta$ in a series of sines of multiples of θ.

(b) If $\sin(A + iB) = x + iy$ prove that $\frac{x^2}{\cosh^2 B} + \frac{y^2}{\sinh^2 B} = 1$ and $\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1$.

(c) Sum to infinity the series $c \sin\alpha + \frac{c^2}{2}\sin 2\alpha + \frac{c^3}{3}\sin 3\alpha + ...$

(2 × 15 = 30)