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B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, NOVEMBER 2015

First Semester

Complementary Course—DIFFERENTIAL CALCULUS AND TRIGONOMETRY

(Complementary Course for Physics/Chemistry/Petrochemicals/Geology, Food Science and Quality Control/Computer Maintenance and Electronics)

[2013 Admission onwards]

Time : Three Hours Maximum : 80 Marks

Part A (Short Answer Questions)

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Find $\lim_{x \to 5} \frac{x-5}{x^2-25}$.

2. If $g(t) = \frac{1}{t^2}$. Find $\frac{dy}{dt}(\sqrt{3})$.

3. Define the average rate of change of $y = f(x)$ with respect to x over the interval $[x_1, x_2]$.

4. State Rolle's theorem.

5. Define a critical point of a function defined on a domain D.

6. State the extreme value theorem.

7. If $f(x,y) = \ln(x+y)$. Find $\frac{\partial f}{\partial y}$.

8. State the mixed derivative theorem for partial derivatives.

9. Express $\cos x - i \sin x$ in terms of exponential function.

10. What is the period of $\sinh(x + yi)$?

(10 × 1 = 10)

Part B (Brief Answer Questions)

Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. Find $\lim_{h \to 0} \frac{\cosh - 1}{h}$.

Turn over

12. Show that the function $y=\sqrt{x}$ is not differentiable at $x=0$.

13. Does the curve $y=x^4-2x^2+2$ have any horizontal tangent ? If so, find the point at which such a tangent occur.

14. Find the point C of mean value theorem for the function $f(x)=1-x^2$ in $0\le x\le 2$.

15. Find the absolute extrema values of $g(t)=8t-t^4$ on [-2, 1].

16. Show that the function $k(t)=\frac{1}{1-t}+\sqrt{1+t}-3.1$ has exactly one zero in the interval $(-1,1)$.

17. If $w=x^2+y^2, x=r-s, y=r+s$ then express $\frac{\partial w}{\partial s}$ in terms of r and s.

18. If $f(x,y)=x\cos y+ye^x$. Find $\frac{\partial^2 f}{\partial y\partial x}$ at (1, 3).

19. Find f_{yxz} if $f(x,y,z)=1-2xy^2z+x^2y$.

20. Find the real part of the expression $\cosh(\alpha+\beta i)$.

21. Prove that $\cosh^2 y-\sinh^2 y=1$.

22. Define $\sin x$ and $\cos x$ interms of exponential functions and verify the result :

$\cos(x-y)=\cos x+\cos y+\sin x\sin y.$

$(8\times 2=16)$

Part C (Short Essay Questions)

Answer any **six** *questions.*
Each question carries 4 marks.

23. Let $\lim_{x\to c} f(x)=\text{L}$ and $\lim_{x\to c} g(x)=\text{M}$. Prove that $\lim_{x\to c}(f(x)+g(x))=\text{L}+\text{M}$.

24. State the sandwich theorem, using this find :

    (a) The horizontal asymptote of the curve $y=2+\frac{\sin x}{x}$.

    (b) Find $\lim_{\theta\to 0}\sin\theta$.

25. If $x=t+\frac{1}{t}, y=t-\frac{1}{t}$, find $\frac{d^2y}{dx^2}$ as a function of t.

26. State the first derivative test for the monotonic function and using this, find the critical point of f if $f'(x) = (x-1)(x+2)(x-3)$ and the intervals in which the function is increasing or decreasing.

27. For what values of a, m and b does the function :

$$f(x) = \begin{cases} 3, x = 0 \\ -x^2 + 3x + a, 0 < x < 1 \\ mx + b \; 1 \le x \le 2 \end{cases}$$

satisfy the hypothesis of the mean value theorem on the interval [0, 2].

28. If resistors R_1, R_2 and R_3 ohms are connected in parallel to make an R–ohm resistor, the values of R can be found from $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$. Find the value of $\frac{\partial R}{\partial R_2}$ when $R_1 = 30$, $R_2 = 45$, $R_3 = 90$ ohms.

29. Give a formula for implicit differentiation in terms of partial derivatives and use it to find ${dy}/{dx}$ if (a) $y^2 - x^2 - \sin xy = 0$; (b) $xe^y + \sin xy + y = 0$.

30. Express $\frac{\sin 6\theta}{\sin \theta}$ in a series of descending powers of $\cos\theta$.

31. Sum to infinity the series : $\frac{1}{2}\sin\alpha + \frac{1}{2}\cdot\frac{3}{4}\sin 2\alpha + \frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\sin 3\alpha + \ldots$

(6 × 4 = 24)

Part D (Essay Questions)

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) Show that the point (2, 4) lies on the curve $x^3 + y^3 - 9xy = 0$ then find the tangent and normal to the curve at (2, 4).

(b) Find a parametrization for the line segment with end points (– 2, 1) and (3, 5).

33. (a) State the first derivative test for local extrema.

(b) Find the critical points of $f(x) = x^{4/3} - 4x^{1/3}$ identify the intervals on which f is increasing and decreasing.

Turn over

(c) Find the position of a body at time t if it is falling freely with initial velocity V (0) = – 3 from a height S (0) = 5 m.

34. (a) Let $f(x,y) = \begin{cases} 0, xy \neq 0 \\ 1, xy = 0 \end{cases}$ then :

(i) Find the limit of f as (x, y) approaches to (0, 0) along the line $y = x$.

(ii) Prove that f is not continuous at the origin.

(iii) Show that both the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exist at the origin.

(b) Express $\partial w/_{\partial r}$ and $\partial w/_{\partial s}$ in terms of r and s if $w = x + 2y + z^2, x = r/_s, y = r^2 + \ln S\ Z = 2r$.

35. (a) Expand $\cos^5\theta\sin^7\theta$ in a series of sines of multiples of θ.

(b) Prove that $\tanh(\alpha+\beta) = \frac{\tanh\alpha + \tanh\beta}{1 + \tanh\alpha\ \tanh\beta}$.

(c) Prove that $\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y$.

(2 × 15 = 30)